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CAPITAL ACCUMULATION AND TECHNICAL CONDITIONS ALONG SUSTAINABLE GROWTH PATHS

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Abstract

The paper deals with the conditions of capital accumulation that can guarantee a sustainable pollution level and, consequently, a growing carrying capacity of the ecosphere for humankind. It follows an approach of ecological economics, utilising laws and models derived from the ecological sciences to model some aspects of growing economic systems and their interaction with "other" ecological systems. The main task of this paper is to demonstrate that, under particular conditions, the earth's carrying capacity for humankind can grow over time as the economic output does, and to illustrate under what conditions it is theoretically possible. Being this kind of analysis a very hard task, the paper will address the question utilising simplified economic models, linking some simplified ecological assumptions to a multisectorial growth model, with production of goods, "bads", and pollution abatement services.

Key words: capital accumulation – population growth – carrying capacity – economic growth – limits to growth – pollution abatement .

JEL Classification: Q560, Q570, O410, J100, O130.

1. Introduction

Our present objective is to identify the conditions of capital accumulation that can guarantee a sustainable pollution level and, consequently, a growing carrying capacity of the ecosphere for humankind. Hence, we will follow a typical approach of ecological economics, utilising laws and models derived from the ecological sciences to model some aspects of economic systems and their interaction with "other" ecological systems. The *carrying capacity* of an ecosystem for a living species is, indeed, an ecological concept, denoting the maximum number of individuals which the ecosystem can support by means of the resources it generates for them. It is also a very "Malthusian" concept, easily connectible to the topic of the existence of natural limits to economic growth. And it is finally a Darwinian and evolutionary concept, because it is the basis of natural selection and dialectic between a biological population, with its variety of individuals and their survival strategies, and its natural environment, viewed as a dynamic system that supplies a periodical flow of resources

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to reproduce itself over time. The sustainable pollution level, in turn, will be here dealt with utilising ecological definitions, as, for example, the quantity consistent with the assimilation capacity of natural ecosystems.

The main aim pursued in this paper is to demonstrate that, under particular conditions, the earth's carrying capacity for humankind can grow over time as the economic output does. A positive population growth rate can be the effect of a continuous shifting of the carrying capacity upwards, produced by capital accumulation and technological progress. This argument can explain the exponential, and sometimes hyperexponential, population growth of the last two centuries as a consequence of the enlargement of the effective human environment produced by economic growth, and not as the exogenous cause of a growing human pressure on the natural environment, as some ecologist literature assumes. Humankind, thanks to capital accumulation and technological change, progressively intercepts the flows of materials and energy previously used by other living species or geophysical and geochemical processes, to satisfy its growing social needs. So, technological change and capital accumulation can be considered as a specific and innovative evolutionary adaptation strategy of the human species, which has begun to adapt the environment to its evolving needs instead of adapting the bodies and physiology of its members to a changing context. From this point of view, technological progress and capital accumulation are a particular kind of *K-strategy*, being a way to increase the natural *carrying capacity*. Obviously, in a finite world this is not possible for ever, and is therefore doomed to fail. Yet, so far, the process has run for more than two hundred years and could probably still go on for some centuries. The principal question is under what conditions and with what effect on the welfare of humankind.

If technical progress is endogenous and connected to capital accumulation, output growth can plausibly proceed without the brakes of decreasing returns, until all the materials and energy of the planet are assigned to satisfy human needs, or until new kinds of "bads", by-produced in traditional production activities, like pollution, increasingly contrast the gains of economic growth. But also this last effect could be contrasted by new production processes oriented to transform "bads" into goods with increasing returns, within the limits of the energy flows passing through the planet.

Of course, this kind of analysis being a very hard task, we will address the question utilising simplified economic models, linking some simplified ecological assumptions to multisectoral growth model, with production of goods, "bads", and pollution abatement services. In particular, we will try to connect a multisectoral model with pollution abatement activities elaborated by Leontief in the seventies with the analysis by vertical integrated sectors conceived by Pasinetti (1973, 1980), applying the structural dynamics conditions of the latter to the former.

Input-Output models present two useful features for an evolutionary approach. They use fixed technical coefficients, which can be considered proxies of production routines that can change by means of innovation (Nelson, Winter, 1973), and have a typical mesoeconomic analytical approach (van den Bergh, Gowdy, 2000; van den Bergh et al., 2007)). The model presented here has very peculiar assumptions, but we hope it can contribute to the search for «concrete replies to some of the fundamental factual questions that should be asked and answered before a practical solution can be found to problems raised by the undesirable environmental effects of modern technology and uncontrolled economic growth» (Leontief, 1970). It also has the merit, like all the input-output models (Wixted et al., 2006), of permitting empirical implementation if the available statistical data became suitable, as recent developments of NAMEA in many countries lead to hope (UN et al., 2003).

2. Production of commodities by means of waste

Production process affects the environment in two ways: extracting resources and releasing waste. We will disregard the former concentrating on the latter.

Waste can generally be processed by natural ecosystems, but to a limited extent. Thus, releasing waste into the environment reduces human well-being when it exceeds the environmental assimilative capacity, damaging natural goods and human health. In this way waste becomes pollution or, from a main stream economic point of view, negative externalities.

In fact, waste is a by-product of regular consumption and production activities, even though it is undesirable. In particular, the waste of production is a form of intrinsic joint products of ordinary goods (Lager, 1999).

Joint production is generally characterised by multiple output generated from only one process (Hosoda, 1998). From a thermodynamic point of view, every production process is a transformation of some kinds of materials and energy into other kinds of materials and energy. The fundamental social fact that a part of new materials are more useful to human beings than the original ones is no matter for nature. Thus, the production process of any good and the technically connected emissions of waste are a particular kind of intrinsic joint production, like mutton-wool or most the chemical production processes, with the only particular feature that one or more of the joint products are useless or even dangerous for human health and environmental safety and quality.

The possibility of valueless joint products, destined to be thrown away, was once considered by Adam Smith, while Karl Marx explicitly dealt with waste as joint products of desired commodities (Kurz, 1986; Hosoda, 1998). Marx, in particular, defined industrial waste *excrements of production*, such as issue of the natural circulation of matter between human society and nature, and was optimist about the chance of their recycling because of the raising of the price of raw materials and progress of chemistry (Marx, 1909).

More recently, this issue was dealt with by Wassily Leontief, who incorporated waste in the context of a conventional multisectoral production model (Leontief, 1970).

As joint products, waste is strictly proportional to the use of the quantities of some inputs, and then, of the other outputs of the production process. The release of carbon dioxide in the atmosphere, for example, shows a technical relation with the quantity of fuel burned to produce energy or mechanical work in most of the production processes. Consequently, in a context of a given technology, and if we assume constant returns to scale, the ratio between each commodity and each waste is given by a constant emission coefficient.

Input-output models connect the level of output of each productive sector to the level of activities in all the other sectors.

Technical linkages between quantities of undesirable and desirable products can be described by means of technical coefficients conceptually similar to those used to model the interdependence among the economic outputs and inputs.

In accordance with Leontief (1970), let us consider a simple economic system with n productive branches and one final sector. Each of the former utilises a part of its own product and provides the rest as inputs to the other productive sectors and to the final one. The final sector is supplying labour services and demanding consumption and investment goods.

The equilibrium conditions for the quantities in this system can be expressed as follows

$$[\mathbf{q}, l]' = \mathbf{A}[\mathbf{q}, l]' \qquad [2.1]$$

where

$\boldsymbol{\omega} = [\mathbf{q}, l]'$ is the column vector compounded by the vector $\mathbf{q}$ of the n produced commodities and the scalar l, denoting the total labour services available in the system at the time t.

Associated with this equation system there is a set of equation describing the total level of waste emissions determined by the production quantities. In fact, if we denote w_{ij}, $i = 1, \dots, m$, the emission coefficient measuring the emission of waste i per unit of commodity j, we can write

$$\mathbf{w} = \mathbf{W}\mathbf{q} \qquad [2.2]$$

where

$\mathbf{w}$ is the vector of the m total waste emissions;
$\mathbf{W} = \lfloor w_{ij} \rfloor$ is the $m \times n$ matrix of the emission coefficients.

Thus, we can model the whole joint production process in the following way

$$\begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{W} & \mathbf{O} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{w} \end{bmatrix} + \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix} \qquad [2.3]$$

where

$\begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{w} \end{bmatrix}$ is the vector of joint outputs;

$\begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{W} & \mathbf{O} \end{bmatrix}$ is the matrix of technological parameters, whose partitions are the already defined matrix $\mathbf{A}$ and $\mathbf{W}$ and two null submatrices with an appropriate number of rows and columns to make the technological matrix an $(n + m)th$ order one;

$\begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$ is the vector of exogenous variables, in which **f** is the vector of final demands.

If the production of goods is strictly joint with the production of bads, the society can only follow two strategies: to choose the best mix of goods and bads or, copying nature, to implement the transformation of waste into resources or into harmless materials. This latter solution however implies a choice between two different uses of production factors and does not completely exclude a decision about an optimal level of waste emission. Thus, the pollution problem is a problem of social product composition.

3. A multisectoral model with pollution abatement services

Following Leontief, now we can model a multisectoral economic system with pollution abatement activities.

Let us consider an economy consisting of $n+m+1$ compartments, where n is the number of industries producing a single commodity, m the number of industries transforming a single waste in a non dangerous substance, and the last compartment is related to labour, with total available workers in each production period treated as output and their consumption of goods treated as inputs. Full employment is assumed as well, so that in each period the sum of workers utilised in all industries equals total workers. In accordance with most of the input-output models, we will also assume constant returns to scale (Pressman, 1998).

Moreover, we will assume the economy is basic. As usual, this means that all commodities enter directly or indirectly into all the processes of production and pollution abatement; but, in our case, it also means that, for institutional reasons, like command and control policies, all abatement processes have to be activated in all the production and pollution abatement processes.

Under all these assumptions, production can be represented by the following equation system

$$\boldsymbol{\omega}^* = \mathbf{A}^* \boldsymbol{\omega}^* \qquad [3.1]$$

where

$\boldsymbol{\omega}^* = [\mathbf{q}, \mathbf{a}, \ell]'$ is the vector composed by the vector $\mathbf{q}$ of the n commodities produced in the economic system, the vector $\mathbf{a}$ of the m pollution abatement services and the scalar ℓ, always denoting the available total quantity of labour services;

$\mathbf{A}^* = \begin{bmatrix} [a_{ij}] & [\alpha_{is}] & [c_i + i_i] \\ [w_{rj}] & [\rho_{rs}] & [\gamma_r] \\ [\ell_j] & [\lambda_s] & h \end{bmatrix}$ is the matrix of constant and nonnegative technical coefficients,

in accordance with the following legend

- $[a_{ij}]$ is the $n \times n$ submatrix of technical coefficients denoting the input of commodity i consumed in the production of a unit of commodity j;
- $[\alpha_{is}]$ is $n \times m$ submatrix of technical coefficients denoting the input of commodity i consumed in the abatement activity of one unit of waste s.
- $[w_{rj}]$ is the $m \times n$ submatrix of the emission coefficients of waste r by unit of commodity j;
- $[\rho_{rs}]$ is the $m \times m$ submatrix of the emission coefficients of waste r by abatement of one unit of waste s;
- $[\ell_j]$ is the $1 \times n$ submatrix of labour coefficients in production of commodities;
- $[\lambda_s]$ is the $1 \times n$ submatrix of labour coefficients in abatement processes;
- $[c_i + i_i]$ is the $n \times 1$ submatrix of the sum of consumption and investment coefficients;

$[\gamma_r]$ is the $m \times 1$ submatrix of standard coefficients, denoting the per capita levels of waste socially endured;
h is the consumption coefficient for final consumption of labour services.

This kind of economic system would usually be said to be viable if it could reproduce itself, barring the subsistence conditions of workers (Kurz, Salvadori, 1997; Bidard, 1998); that is, if there was a vector $\hat{\mathbf{q}}$ such that

$$\hat{\mathbf{q}} \geqq \hat{\mathbf{A}}\hat{\mathbf{q}} \quad [3.2]$$
$$\hat{\mathbf{q}} \geq \mathbf{0} \quad [3.3]$$

where

$\hat{\mathbf{q}} = [\mathbf{q}, \mathbf{a}]'$ is the vector composed only by the vector $\mathbf{q}$ of the n commodities and the vector $\mathbf{a}$ of the m abatement services;
$\hat{\mathbf{A}} = \begin{bmatrix} [a_{ij}] & [\alpha_{ij}] \\ [w_{ij}] & [\rho_{ij}] \end{bmatrix}$ in which the symbols have the same meaning as the corresponding ones in [3.1].

In fact, this is a strictly technical definition of viability, in which attention is exclusively restricted to the reproduction conditions of commodities by means of themselves. In this paper we prefer distinguish this kind of conditions, which we will define technical viability, from a broader social viability, in which even the social subsistence is considered. The latter is a very multiform concept that could be defined in a lot of different ways (Picchio, 1998), but here we prefer to define it simply as basic needs that have to be satisfied by production activities in order to reproduce the society and individuals that constitute it, in their historically determined conditions. Thus, the system is socially viable if and only if

$$\hat{\mathbf{q}} \geqq (\mathbf{A} + \bar{\mathbf{c}}\mathbf{l}')\hat{\mathbf{q}} \quad [3.4]$$
$$\hat{\mathbf{q}} \geq \mathbf{0} \quad [3.5]$$

where

$\bar{\mathbf{c}} = [\bar{c}_1, \ldots, \bar{c}_n, \bar{\gamma}_1, \ldots \bar{\gamma}_m]'$ is the vector of subsistence coefficients;
$\mathbf{l} = [\ell_1, \ldots, \ell_{n+m}]'$ is the vector of labour coefficients;

or, equivalently, if and only if

$$\boldsymbol{\omega}^* \geqq \mathbf{A}^{**}\boldsymbol{\omega}^* \quad [3.6]$$
$$\boldsymbol{\omega}^* \geq \mathbf{0} \quad [3.7]$$

where

$$\mathbf{A}^{**} = \begin{bmatrix} [a_{ij}] & [\alpha_{ij}] & [\bar{c}_i] \\ [w_{ij}] & [\rho_{ij}] & [\bar{\gamma}_i] \\ [\ell_j] & [\lambda_j] & h \end{bmatrix}$$

Necessary and sufficient condition for [3.6] and [3.7] is

$$\begin{array}{l} 1-a_{11} > 0 \\ \left\| \begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \right\| > 0 \\ \cdots\cdots\cdots\cdots \\ \left| \mathbf{I} - \mathbf{A}^{**} \right| \geq 0 \end{array} \qquad [3.8]$$

This means that in an economic system with pollution abatement activities, in addition to usual viability conditions, it also has to be satisfied the condition that waste directly and indirectly produced to depollute one unit of the same waste has to be less than one unit (Hawkins, 1948; Hawkins, Simon, 1949; Weale, 1984; Kurz, Salvadori, 1997).

4. A dynamic model with evolving carrying capacity and pollution abatement services

The previous static analysis was conducted following a multisectoral input-output approach. But for a dynamical analysis it can be useful to pass to a vertically integrated one. The passage from our multisectoral model to a vertically integrated one can easily be realised by means of a simple linear transformation. In accordance with Pasinetti (1981), the first step is to open the equation system [3.1] with reference to the final sector, as follows

$$\hat{\mathbf{q}} = \hat{\mathbf{A}}\hat{\mathbf{q}} + \mathbf{f} \qquad [4.1]$$

where

$\hat{\mathbf{q}} = [\mathbf{q}, \mathbf{a}]'$ is the vector composed only by the vector **q** of the n commodities and the vector **a** of the m pollution abatement services;

$\mathbf{f} = [(\mathbf{c} + \mathbf{i})', \boldsymbol{\gamma}']'$ is the vector of final variables, dealt with as exogenous with respect to the equation system.

The second step is to resolve this equation system for the vector $\hat{\mathbf{q}}$, as follows

$$\hat{\mathbf{q}} = (\mathbf{I} - \hat{\mathbf{A}})^{-1}\mathbf{f} \qquad [4.2]$$

where

$\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}=\begin{bmatrix}\left[\widehat{a}_{ij}\right] & \left[\widehat{\alpha}_{ij}\right]\\ \left[\widehat{w}_{ij}\right] & \left[\widehat{\rho}_{ij}\right]\end{bmatrix}$ is the $(n+m)\times(n+m)$ matrix of activation coefficients, in which

- $\widehat{a}_{ij}$ is the multiplier that determines the total output of the commodity i necessary to produce one unit of the commodity j;
- $\widehat{\alpha}_{ij}$ is the multiplier that determines the total output of the commodity i necessary to treat one unit of the waste j.
- $\widehat{w}_{ij}$ is the multiplier that determines the total quantity of the waste i society has to treat to produce one unit of the commodity j;
- $\widehat{\rho}_{ij}$ is the multiplier that determines the total quantity of the waste i society has to treat to clear the environment of one unit of the waste j;

The third step is to post-multiply the transpose of $\left(\mathbf{I}-\widehat{\mathbf{A}}\right)^{-1}$ by the vector of labour coefficients $\mathbf{l}=\left[\ell_1,\ldots,\ell_n,\lambda_1,\ldots\lambda_m\right]'$, to obtain the vector $\mathbf{v}$, as follows

$$\mathbf{v}=\left[\left(\mathbf{I}-\mathbf{A}\right)^{-1}\right]'\mathbf{l} \qquad [4.3]$$

Hence $\mathbf{v}$ is the vector of total labour quantities that society has directly and indirectly to utilise producing each unit of final goods and varying the final pollution standards by a unit. Following Pasinetti (1981), let us also assume that, at each time *t*, the economic system is utilising a capital stock vector $\mathbf{K}(t)=\left[K_1 \quad \cdots \quad K_n\right]'$, where K_i denotes the capital stock utilised in the productive or depolluting sector i and measured in units of production capacity of its final commodity or service. These capital stocks are a heritage of previous periods of time, being the sum of all net investment over the past.

At each time there is also another stock, fundamental in determining labour availability and consumption and investment demand: population. It is usually dealt with as a result of exogenous demographic dynamics. But in this paper we will assume it endogenously determined by means of an economic interpretation of the ecological concept of *carrying capacity*. Indeed, we will deal with the whole planet as a single ecosystem, connecting its *carrying capacity* for humankind with two economic concepts: social subsistence and labour productivity (Scarano, 2008).

In ecology, the *carrying capacity* for a living species is usually defined as the maximum number of individuals which an ecosystem can support by means of the resources it generates for them. The social subsistence, as previously said, can in turn be defined as basic needs that have to be satisfied by production activities in order to reproduce the society and individuals that constitute it, in their historically determined conditions. On account of its historical nature, this kind of subsistence can be dealt with as varying in accordance with any exogenous function of time. Labour productivity, instead, is considered, in a more usual way, as the average product by worker. Thus, it is evident that labour productivity determines the availability of goods fit to satisfy the social subsistence needs, and so it also determines the carrying capacity for social individuals.

Hence, following this line of reasoning, in a single product model we can assume that

$$N_{\max}(t) = \frac{C(t)}{c(t)} \qquad [4.4]$$

where

$N_{max}(t)$ is the *carrying capacity*;
C(t) is the aggregate potential consumption at time *t*;
c(t) is the function of the necessary *per capita* consumption or *per capita* social subsistence at the same moment (Scarano, 2008).

In a multisectoral model the definition of the *carrying capacity* must become more complicated. Indeed, in this context there are n commodities and n *per capita* social subsistence quantities. Thus, there are two possibilities in defining $N_{max}(t)$. We can assume it is determined by the relatively a scarcer good, working as limiting factor, as follows

$$N_{\max}(t) = \min \frac{C_i(t)}{\bar{c}_i(t)} \qquad (i = 1, \ldots, n): \qquad [4.5]$$

Otherwise, we can assume there is some kind of social coordination, adapting productions of goods to their demands at each time. Whether coordination is realised by market clearing or by a central planner it is not on issue here. So, the effective consumption vector will be a scalar multiple of *per capita* social subsistence vector, as follows

$$\mathbf{C}(t) = \lambda \bar{\mathbf{c}}(t) \qquad [4.6]$$

where

$\mathbf{C}(t) = [C_1(t) \quad \cdots \quad C_n(t)]'$ is the vector of the available consumption quantities for the n goods produced in the economy at time t;
$\bar{\mathbf{c}}(t) = [\bar{c}_1(t) \quad \cdots \quad \bar{c}_n(t)]'$ is the vector of *per capita* social subsistence quantities for the n goods produced in the economy at time t;
λ is a scalar.

Doing so, we have normalised consumption for social subsistence and $\mathbf{C}(t)$ is collinear with $\bar{\mathbf{c}}(t)$. Thus $N_{max}(t)$ can be determined as follows

$$N_{\max}(t) = \frac{|\mathbf{C}(t)|}{|\bar{\mathbf{c}}(t)|} = \lambda \qquad [4.7]$$

$$|\bar{\mathbf{c}}(t)| = 1 \qquad [4.8]$$

where

$|\mathbf{C}(t)|$ is the length or norm of the vector $\mathbf{C}(t)$;

$|\bar{\mathbf{c}}(t)|$ is the length or norm of the vector $\bar{\mathbf{c}}(t)$.

If we now multiply each member of [] by a vector of given price for the n goods at the time t, we obtain

$$\mathbf{p}(t)\mathbf{C}(t) = \mathbf{p}(t)\lambda\bar{\mathbf{c}}(t) = \lambda\mathbf{p}(t)\bar{\mathbf{c}}(t) \qquad [4.9]$$

Hence, N_{max} (t) computed in terms of consumption and subsistence quantities normalised coincide with [4.4].

We can now assume that population grows over time if ***C***(t) grows faster than **c**(t).

The labour force is assumed to be proportional to the total population as follows

$$\ell(t) = \mu(t)\nu(t)N(t)$$

[4.10]

where $\mu(t)$ is the activity rate of population and $\nu(t)$ is the normal labour time by worker.

Now, production in terms of vertically integrated sectors can be represented by the following equation system

$$\begin{bmatrix} -1 & \cdots & \cdots & \cdots & \cdots & \cdots & \bar{c}_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & -1 & \cdots & \cdots & \cdots & \bar{\gamma}_m \\ a_{k_1 1} & \cdots & \cdots & -1 & & & i_{k_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & & & & a_{k_{n+m} n+m} & -1 & i_{k_{n+m}} \\ \ell_1 & \cdots & \ell_{n+m} & \ell_{k_1} & \cdots & \ell_{k_{m+m}} & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ d_m \\ k_1 \\ \vdots \\ k_{n+m} \\ \ell(t) \end{bmatrix} = \mathbf{0}$$

[4.11]

In order for the equation system [4.11] to be able to have non-trivial solution, the two following conditions have to be satisfied.

$$\sum_{i=1}^{n} l_i c_i + \sum_{i=1}^{m} \lambda_i \gamma_i + \sum_{i=1}^{n+m} l_{k_i} i_k + \sum_{i=1}^{n+m} (1/T_1) l_{k_i} i_k = 1$$

[4.12]

$$K_i = \hat{q}_i \qquad (i = 1, \ldots, n, n+1, \ldots n+m)$$

[4.13]

Of course, the former is a macroeconomic condition, while the latter is a sector condition.

Given technology, over the time the previous condition [4.12] becomes

$$\dot{K}_i(t) = \dot{\hat{q}}_i(t) \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.14]

that is, production capacity in the sector i has to grow as its demand.

Each demand for capital good is split into two parts, as follows

$$k_i(t) = (1/T_i)c_i\ell(t) + i_i\ell(t) \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.15]

where $(1/T_i)c_i\ell(t)$ is depreciation of capital and $i_i\ell(t)$ is new investment in the sector i.

Since

$$i_i\ell(t) = \dot{K}_i(t) \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.16]

then

$$i_i\ell(t) = \dot{\hat{q}}_i(t) = \hat{c}_i\,\dot{\ell}(t) \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.17]

This means that the sector equilibrium condition implies that investment has, over time, to equal the variation rate of demand, determined in turn by the variation of labour force.

Substituting [4.12] into [4.19], we have

$$i_i\mu(t)\nu(t)N(t) = c_i\mu(t)\nu(t)\dot{N}(t) \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.18]

that implies

$$i_i = \frac{\dot{N}(t)}{N(t)}c_i = n(t)c_i \qquad (i = 1,\ldots,n,n+1,\ldots n+m)$$
[4.19]

where $n(t)$ is the growth rate of population given by the following expression (Scarano 2008)

$$n(t) = n_i\left(1 - \frac{N(t)}{C(t)/c(t)}\right) = n_i\left(1 - \frac{c(t)}{(1-s)\,y(t)}\right)$$
[4.20]

where s is the average propensity to save and *y(t)* is per capita output or the labour productivity.

So, [4.19] is the dynamic equilibrium condition for capital accumulation. In fact, it is the multisectoral version of Harrod-Domar equation (Pasinetti, 1981). It is evident that, under our assumptions, it is defined by the intrinsic growth rate of population, but also by the levels of social subsistence, propensity to save and labour productivity.

This is a co-dynamic condition, but it can also be seen as a co-evolution constraint, because the performance of pollution abatement activities contributes to produce environmental conditions of commodities sectors, in terms of waste or bads that can reduce the productivity of the economic system, as well as diminishing utility of individuals (van den Bergh, Stagl, 2003; van den Bergh et al., 2007).

5. Conclusions

Under the hypotheses of a Leontief technology, with viability conditions respected, disregarding the problems connected with scarce natural resources, the economic system can grow without any limit, also in the presence of joint production of waste, if there are viable pollution abatement processes. In our simplified model growth rate is substantially determined by investment coefficients. The former can obviously be considered as directly determined by propensity to save and this, in the context of a classical approach, can be connected to the rate of profit. The model presented can be utilised to resolve the problem of prices as the dual of that of the quantities. But this is another story.

Here we can observe that in the real world, on the waste front, the environmental sustainability problem is connected to the problem of achieving the right composition of social product. In a free market economy, this composition could be adequate only if the rate of profit was uniform between commodities sectors, on one side, and abatement services, on the other. The market failure in guaranteeing this uniformity is the principal cause of disproportion in investment plans and consequently in capital stock composition. But this is a social limit, and not a natural one.

The real natural limit to this process is probably on the front of natural resources – but not of all of them. In fact, technological change has continuously substituted old materials with new ones, in a permanent process of innovation. The real problem could lie entirely on the energy front, because today it is directly and indirectly connected to solar radiation, which so far remains necessary and unreproducible. But also on this front, innovation can play a strategic role in defining a sustainable development path over time. Decreasing emission coefficients in commodities production as well as decreasing input coefficients in abatement activities can, indeed, improve sustainability on the front of resources and energy for a long time.

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