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PROSPECT THEORY AND SELF-FULFILLING MARKET SENTIMENTS

Giuseppe Ciccarone - Francesco Giuli - Enrico Marchetti

Prospect Theory and self-fulfilling market sentiments[1]

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Abstract: In this paper we present a novel channel through which the volatility of the monetary/financial sector affects the instability of the real macroeconomic variables originated by self-fulfilling market sentiments. To this aim, we insert some elements of Prospect Theory in the preferences of agents living in an overlapping generations economy where consumers' heterogeneity and firms' imperfect information on the level of aggregate demand allow market sentiments to affect the equilibrium path of the economy. In this environment, greater heterogeneity in the household's narrow framing parameter favour the emergence of self-fulfilling equilibria by exacerbating the coordination problem generated by a pair-wise matching process in the labor market. Furthermore, higher volatility of the money market, by increasing the effect of Prospect Theory on households' choices, makes the signal upon which firms form their demand schedules noisier; this, in its turn, generates greater variability in market sentiments and hence in real economic activity.

JEL Classification: D840; E030; E320.
Keywords: Prospect theory; Self-fulfilling equilibria; Sentiments; Behavioral macroeconomics.

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1 Introduction

In the recent economic debate, the role and relevance of the link between the emergence of financial instability and the increase in macroeconomic volatility has often been considered as an important explanatory element of major developments, such as the great depression and the possible ending of the Great Moderation (see, e.g., Clark, 2009). At the same time, the potentially fruitful attempt to integrate this finance-macroeconomy interaction with the view that income fluctuations may be sentiment driven, which has progressively attracted a renewal of interest in the literature, has not yet been fully carried out.

The aim of this paper is to propose an additional channel through which the volatility of the monetary/financial sector can contribute to the overall instability of the economy originated by self-fulfilling market sentiments. The mechanism generating this outcome is based on the interaction between two main elements which have gained increasing relevance in the recent theoretical debate on business cycles and economic crisis[2].

The first of these elements is the Keynesian idea of animal spirits as a major source of aggregate economic fluctuations, which has been recently incorporated into an innovative class of models by Angeletos and La'O (2013) and Benhabib et al. (2012; 2013 and 2015). The general idea behind this approach is that imperfect information, or communication, on the level of some aggregate variable (e.g., demand or income) can create room for sunspot-like random variables to affect the equilibrium path of the economic system. In the spirit of Lucas's (1972) island model, when agents take their decisions before observing these aggregate variables, their choices may come to depend upon some noisy and imperfect signal about the same variables. This implies that extrinsic uncertainty (related to the conjectures made on aggregate variables) may affect decisions and that multiple sentiment-driven equilibria, which are stochastic in their nature, may arise under rational expectations even in the absence of intrinsic uncertainty (stochastic shocks to the economy's fundamentals). Furthermore, these sentiment driven equilibria do not rely, for their existence, upon arbitrary randomization devices applied to a unique deterministic equilibrium, which was a typical feature of previous models of sunspot fluctuations. Even though one possible cause of economic fluctuations determined by these waves of optimism or pessimism can be traced back to the decentralization of market interaction, modeled by Angeletos and La'O (2013) through a pair-wise matching process, every feature of the economic environment creating obstacles to information and communication flows can potentially give rise to this form of self-fulfilling dynamics.

The second element which has recently attracted special attention in the literature, especially in the light of the great crisis, is the role played by the financial sector, not only in shaping the business cycle, but also in producing wide and abrupt economic changes. In this field, besides the attempt to include financial frictions in standard new Keynesian DSGE models,[3] or to explore the formation of (rational) bubbles in asset prices (as, e.g., in

[2]This paper shares characteristics with a recent strand of the literature aiming to contaminate traditional dynamic equilibrium models with more realistic assumptions about agents' preferences and expectation formation. See, e.g., Gaffeo et al. (2014) and the surveys by Brzoza-Brzezina et al. (2013), and Zhang and Semmler (2009).

[3]See Ciccarone et al. (2014) and the references there contained, and the survey by Woodford (2010).

Martin and Ventura 2012), an important role has been attributed to agents' choices under risk described through deviations from the standard assumption of rational behavior, as depicted by expected utility maximization. Among these "non traditional" choice theories, Tversky and Kahneman's (1992) cumulative Prospect Theory (PT) is the most successful one, as proved by the evidence collected in experimental settings,[4] the data gathered in financial markets (Kliger and Levy, 2009) and in other market environments,[5] as well as PT's ability to solve a number of theoretical and empirical puzzles. As a consequence, PT is now widely adopted in the flourishing field of behavioral finance and starts to make its appearance in macroeconomic modelling.[6]

Here we show that PT elements related to households' consumption-saving choices can create a new theoretical mechanism, not relying on nominal price rigidities, which explains the transmission of volatility from monetary/financial markets to the real macroeconomic variables. On the one hand, monetary variance (i.e., the variance of the money stock, conceived as a financial asset in an overlapping generations - OLG - setting) affects the choices under risk of consumers/savers characterized by loss aversion and a narrow framing attitude. On the other hand, if agents are heterogeneous with respect to some of these behavioral characteristics, imperfect information and communication can create sentiment-driven fluctuations akin to those arising in the model recently presented by Benhabib et al. (2012, 2013 and 2015). More specifically, we reformulate a simple version of this model, which allows for explicit closed-form solutions, through an OLG framework where consumers/workers have heterogeneous preferences and imperfect communication/information is modelled by means of a pair-wise matching process in the labor markets between monopolistic producers and workers. We then include some elements of PT by adding to the households' utility functions a term which describes their perception of gains/losses in financial wealth.[7] A multiplicative parameter attached to the PT utility component, which measures the degree of *framing,* or better of the *narrow focusing illusion*,[8] is made specific to each household in order to explicitly model the heterogeneous importance of the PT elements across agents.

Our main results are the following three. First, for a wide range of parameters' values, in our model economy there exists a self-fulfilling, sentiment-driven, equilibrium. The emergence of such an equilibrium, which requires both heterogeneity in the households' framing attitude and a form of imperfect communication/information, is favoured by a higher heterogeneity in the framing parameter and in the degree of competition in goods markets, as they exacerbate the coordination problem generated by the pair-wise matching process taking place in the labor markets. Second, while agents' deviations from standard rationality are not sufficient *per se* to generate sentiment-driven fluctuations, we show that the more dispersed are these deviations the higher is the volatility of the economy due to sentiment

[4] See, among others, Tversky and Kahneman (1992), Gonzales and Wu (1999), Donkers et al. (2001).

[5] For an overview of these results, see Camerer (2000).

[6] See for instance, Gaffeo et al. (2014).

[7] This is done in a way similar to that proposed by Barberis et al. (2001). See also Ciccarone and Marchetti (2013).

[8] According to Camerer (2005, pp. 130-131), "A crucial ingredient in empirical applications of loss aversion is decision isolation, or focusing illusion, in which single decisions loom large even though they are included in a stream of similar decisions. [...] Therefore, for loss aversion to be a powerful empirical force requires not only aversion to loss but also a narrow focus such that local losses are not blended with global gains".

fluctuations. A higher dispersion increases in fact the variability of the intermediate firms' sentiments on the state of aggregate demand and in this way amplifies the volatility of (real) macroeconomic variables. Third, as instability in the money market affects agents' choices due to the presence of a loss aversion component affecting their evaluations of the prospect of possible gain and losses in financial wealth, when monetary/financial instability increases the signal intermediate firms receive on the state of the economy and upon which they form their labor demand schedules become noisier. This, in its turn, allows for a greater variability in market sentiments and hence in real economic activity. We stress the fact that this novel transmission channel of nominal volatility to the real economy is entirely independent from the presence of nominal rigidities in price setting.

The paper is structured as follows. The next section introduces the model and describes the behavior of households and firms acting in the economy. Section 3 illustrates the information structure of the model, which characterizes the decentralized trade taking place in the labor markets, and determines the households' demand for consumption and supply of labor, as well as the equilibrium output of the intermediate firms. Section 4 derives the macroeconomic equilibrium of the model and Section 5 presents the effects of framing heterogeneity on sentiment-driven fluctuations. Section 6 concludes.

2 The model economy

2.1 Markets and agents

The economy contains four types of markets: (i) one market for a homogeneous consumption good Y; (ii) a continuum of $j \in [0;1]$ markets for the intermediate goods Y_j which are used as inputs in the production of Y; (iii) a continuum of j markets for the labor inputs L_j (which are used in the production of the Y_j); (iv) a market for money M. In this markets operate households, firms and a monetary authority. More in particular, there exists a continuum of $i \in [0;1]$ heterogeneous households, each one endowed with a specific utility function. Households, who buy the final good Y, formulate individual demands for final consumption, C_i, from which the aggregate demand $\int_0^1 C_i di$ is computed, and supply labor input to intermediate producers. More precisely, as the labor markets are characterized by a form of decentralized pair-wise matching to be thoroughly described below, a specific quantity of labor L_i is sold to a specific intermediate firm j. When a match is formed, the intermediate firm j uses this labor input L_i bought from the $i-th$ household in the $j-th$ labor market to produce the intermediate good Y_j. All the intermediate producers use the same technology and are hence homogenous agents (with respect to their fundamentals). A single final producer (an aggregator, or a wholesale firm) buys the intermediate goods Y_j from the monopolistic competitors operating in the j intermediate markets, uses them as inputs in the production of Y and sells this final good to the consumers. Finally, an external authority issues money, M, which is used by household as a storage of value and is the unit of account.

The market for final consumption is competitive and centralized with respect to information flows and distribution: the price of the final good, P, is perfectly observed by both the households and the wholesale firm, and is common knowledge for these agents. In each of the intermediate markets Y_j the incumbent firm operates under monopolistic competition and sets the price P_j of its good; the prices of the intermediate goods are perfectly observed by the wholesale firm. The j labor markets are competitive but decentralized: in each of them trade takes place only after the realization of a random pair-wise matching between an household and an intermediate firm. Finally, the money market is centralized and competitive; the money supply is exogenous and money is used only by the households as a device to transfer wealth over time (between subsequent time periods).

The markets of Y, Y_j and L_j operate simultaneously, but our assumptions on the structure of information flows and on the existing coordination (or lack thereof) among agents allow us to distinguish between two distinct decisional phases. In a first phase, each household i and each firm j "prepare" to trade in a labor market; here the mechanism is decentralized due to a matching process randomly assigning a specific worker i to a specific firm j. As households' preferences are heterogeneous, a firm j cannot know in advance the "type" of household (worker) it will be matched with. Firm j must hence define its plan (its demand for labor) on the basis of a received signal, S_j, on the partner's type and must compute (rational) expectations conditional on this information. Knowing that intermediate firms are all equal, the generic $i-th$ worker does not face an analogous problem. Once the agents are matched, trade in each labor market takes place: the intermediate firms produce the intermediate goods which are sold to the wholesale aggregator. Trade takes then place in the final good market and in the market for money.

The crucial elements of our model are hence the random matching mechanism in the decentralized labor markets and the heterogeneity of households, which imposes each intermediate firm to solve a signal extraction problem (with rational expectations) created by the imperfect knowledge of the worker/household type. The structure of information flows and the centralized-decentralized working of the various markets is detailed in Section 3.

2.2 Households: behavioral elements and heterogeneity

Each household i, which is composed of a single representative agent, is described through an overlapping-generations scheme with a simplified structure: the agent lives for two periods, works when young and consumes when old. In order to finance consumption when old, the young agent uses labor income and profit income from firms' ownership to buy money at time t. This amount of money is then used to buy consumption goods at $t+1$, when the agent becomes old.

As in Lucas's (1972) model, money M_{it} is supplied to household i by an external authority according to the dynamic law:

$$M_{it} = X_t M_{it-1}$$

X_t is an aggregate exogenous monetary shock, which is assumed to be log-normally distributed: $x_t = \ln X_t \sim \mathbf{N}\left(0, \sigma_x^2\right)$, with σ_x^2 interpreted as a parameter that can be affected by the monetary authority's decisions; x_t is i.i.d. and uncorrelated with other stochastic variables which are present in the model. The aggregate money supply is then equal to

$M_t = X_t M_{t-1}$ with $M_t = \int_0^1 M_{it} di$. Equilibrium in the money market is given by:

$$l_{it} = M_{it}; \qquad \int_0^1 l_{it} = M_t$$

where l_{it} is the demand for money of the (young) agent i. We assume a constant population of households, with the distribution of households types unchanging through time.[9]

The agent's i bi-periodal budget constraint is:

$$C_{it+1} P_{t+1} = l_{it} X_{t+1} = X_{t+1} D_{it} \tag{1}$$

where C_{it+1} is consumption when old and:

$$D_{it} = W_{it} N_{it} + \Pi_{it} \tag{2}$$

is the young agent's income. $W_{i,t}$ is the nominal wage rate in the specific labor market in which the agent i sells the labor input N_{it}. Π_{it} is the profit accruing to him/her from the ownership of the shares of the intermediate firm j, which we assume to be handed to the young worker i during the matching process taking place in the labor market. Instead of exogenously assigning shares of different firms to household i, we hence assume that each young worker becomes the owner of the firm s/he is randomly matched with. This assumption allows us to obtain a tractable model which is identical in this respect to Lucas's (1972) one, where each island's worker is the owner of the same island's firm.

The agent i living at time t has the expected utility function:

$$U_{it} = \beta_i^{-1} E\left(\frac{C_{it+1}^{1-\gamma}}{1-\gamma} \left|\Omega_{it}^H\right.\right) - \frac{N_{it}^{1+\psi}}{1+\psi} + \beta_i E\left[v\left(R_{it+1}, R_{\text{ref},it}\right)\left|\Omega_{it}^H\right.\right] \tag{3}$$

where $\gamma \in (0;1)$ is the curvature parameter of the consumption term, E is the expectation operator, Ω_{it}^H is i's information set and $\frac{N_{it}^{1+\psi}}{1+\psi}$ with $\psi > 0$ represents the agent's disutility of working. At time t the agent cares not only about his/her expected consumption level $C_{i,t+1}$, but also about his/her expected real financial wealth $R_{i,t+1}$, as compared to a reference point $R_{\text{ref},it}$. The function $v(\cdot,\cdot)$ synthesizes the PT component in the agent's utility. The agent hence suffers from some form of the narrow framing effect described by Barberis and Huang (2009)**:** when evaluating financial wealth, s/he considers it *per se*, in addition to the expected utility of the consumption it can produce at time $t+1$. The parameter $\beta_i \in (0;\beta_{\max})$,[10] on the one hand, describes agents' heterogeneity and, on the other one, measures the importance of financial gains and losses in the utility function relative to that of consumption *per se*: when β_i decreases, the standard utility of consumption rises and the PT component decreases.

In the money market equilibrium, the agent i's real wealth is equal to:

[9] As it will be shown below, this allows for simple aggregation with respect to agents i.

[10] In the subsequent analysis we will show that, for the model to be tractable, the upper bound $\beta_{\max}$ must be a specific finite value.

$$R_{t+1} = \frac{X_{t+1}M_{it}}{P_{t+1}}$$

His/her reference level of wealth, $R_{\text{ref},t}$ is assumed to be:

$$R_{\text{ref},t} = \frac{M_{it}}{P_{t+1}}$$

$R_{\text{ref},t}$ is then equal to the amount of real asset (money) that would be obtained if no monetary shock occurred, i.e., $X_{t+1} = 1$, or $M_{it+1} = M_{it}$, so that X_{t+1} can be conceived as a stochastic gross rate of return on wealth.

The function $v(\cdot, \cdot)$ is equal to:

$$v = \begin{cases} \left(\frac{X_{t+1}M_{it}}{P_{t+1}} - \frac{M_{it}}{P_{t+1}}\right)^{1-\gamma} & \text{for } \frac{X_{t+1}M_{it}}{P_{t+1}} - \frac{M_{it}}{P_{t+1}} \geq 0 \\ -\eta\left[-\left(\frac{X_{t+1}M_{it}}{P_{t+1}} - \frac{M_{it}}{P_{t+1}}\right)\right]^{1-\gamma} & \text{for } \frac{X_{t+1}M_{it}}{P_{t+1}} - \frac{M_{it}}{P_{t+1}} < 0 \end{cases} \tag{4}$$

and it encapsulates three of the main elements of Tversky and Kahneman's (1992) PT: i) *reference dependence*, i.e., the carriers of utility are wealth gains and losses relative to some reference point; ii) *declining sensitivity*, i.e., the utility function is concave in the domain of gains and convex in the domain of losses; iii) *loss aversion*, i.e., losses are more salient than gains.[11] In particular, the parameter η (which is common to all i) measures the agent's aversion to losses in wealth.

Factoring out $\frac{M_{it}}{P_{t+1}}$ in equation (4) we can write:

$$v = \left(\frac{M_{it}}{P_{t+1}}\right)^{1-\gamma} \begin{cases} (X_{t+1} - 1)^{1-\gamma} & \text{for } X_{t+1} \geq 1 \\ -\eta\left[-(X_{t+1} - 1)\right]^{1-\gamma} & \text{for } X_{t+1} < 1 \end{cases}$$

The agent's choice problem is then:

$$\max_{C_{it+1}, N_{it}} \beta_i^{-1} E\left(\frac{C_{it+1}^{1-\gamma}}{1-\gamma}\left|\Omega_{it}^H\right.\right) - \frac{N_{it}^{1+\psi}}{1+\psi} + \beta_i E_{it}\left(v\left|\Omega_{it}^H\right.\right)$$

$$\text{s.t.} \quad : \quad C_{it+1} = \frac{X_{t+1}P_t}{P_{t+1}}\left(\frac{W_{it}}{P_t}N_{it} + \frac{\Pi_{it}}{P_t}\right)$$

and the first order conditions can be summarized in the following equation:

$$N_{it}^{\psi} = \beta_i^{-1} E\left[C_{it+1}^{-\gamma}\frac{X_{t+1}}{P_{t+1}}W_{it}\left|\Omega_{it}^H\right.\right] + \beta_i E\left(\frac{C_{it+1}^{-\gamma}}{P_{t+1}}W_{it}\begin{cases}\left(1-X_{t+1}^{-1}\right)^{1-\gamma} & \text{for } X_{t+1}\geq 1 \\ -\eta\left[-\left(1-X_{t+1}^{-1}\right)\right]^{1-\gamma} & \text{for } X_{t+1}<1\end{cases}\left|\Omega_{it}^H\right.\right) \tag{5}$$

[11] This representation is in line with Barberis et al. (2001) and Ciccarone and Marchetti (2013). The complete version of PT - i.e., Tversky and Kahneman's (1992) *cumulative* prospect theory - also includes *non-linear weighting of probabilities* (agents facing uncertain situations overweight small probabilities but underweight large ones) and *susceptibility to framing effects* (agents' preferences are influenced by the way a problem is presented). For an exhaustive discussion see, e.g., Wakker (2010).

2.3 The wholesale firm

Following Benhabib et al. (2013), we assume that the competitive wholesale firm, which operates under perfect foresight, produces the final good Y_t after the matching phase in the decentralized labor markets is completed and the individual demand functions for the consumption good are determined. Its production technology is:

$$Y_t = \left[\int Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1 \tag{6}$$

Denoting $P_{j,t}$ the price of the $j - th$ intermediate good Y_{jt}, the wholesale firm chooses the inputs Y_{jt} so as to maximize its profit:

$$\max_{Y_{jt}} P_t \left[\int Y_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} - \int P_{jt} Y_{jt} dj$$

The first order condition is:

$$\frac{P_{jt}}{P_t} = \left(\frac{Y_t}{Y_{j,t}} \right)^{\frac{1}{\theta}} \tag{7}$$

Using the production function (6), we obtain a standard expression for the average price index of the final good: $P_t^{1-\theta} = \int P_{jt}^{1-\theta} dj$.

2.4 The intermediate firms

Recall that in the decentralized trade taking place in the "local" labor market, each household/worker i is matched with an intermediate firm j and obtains the profit of the firm s/he is matched with: $\Pi_{it} = \Pi_{jt}$. Hence we can write:

$$C_{it+1} = \frac{X_{t+1} P_t}{P_{t+1}} \left(\frac{W_{it}}{P_t} N_{it} + \frac{\Pi_{it}}{P_t} \right) = \frac{X_{t+1}}{P_{t+1}} \left(P_{jt} Y_{jt} \right) \tag{8}$$

The intermediate firm j operates with the production function:

$$Y_{jt} = N_{jt} \tag{9}$$

and its profit is hence equal to $\Pi_{jt} = P_{jt} Y_{jt} - W_{jt} N_{jt}$. As the firm is a monopolist in the $j - th$ intermediate good market, it sets P_{jt} or, alternatively, Y_{jt} taking into account the demand function (7) of the wholesale firm. It must also formulate its demand for labor input N_{jt} to be compared with the individual labor supply of the worker it will be matched with. Due to the decentralization of this trade mechanism, firm j cannot anticipate with certainty the value of the wage W_{jt} it will pay and must hence seek to maximize its expected profit, conditional on the available information contained in the information set Ω_{jt}^F.

$$\begin{aligned} \max_{Y_{jt}} \Pi_{jt} &= E\left(P_{jt} Y_{jt} - W_{jt} N_{jt} \middle| \, \Omega_{jt}^F \right) \\ \text{s.t.} &: \; Y_{jt} = N_{jt}; \quad P_{jt} = P_t Y_t^{\frac{1}{\theta}} Y_{j,t}^{-\frac{1}{\theta}}; \end{aligned}$$

The resulting first order condition:

$$\left(1-\frac{1}{\theta}\right) N_{j,t}^{-\frac{1}{\theta}} E\left(P_t Y_t^{\frac{1}{\theta}} \middle| \Omega_{jt}^F \right) = E\left(W_{jt} \left| \Omega_{jt}^F \right.\right) \tag{10}$$

begets the standard equality between expected marginal revenue and expected marginal cost.

3 Information structure and decentralized trade

As hinted at above, we assume that each time period t is divided into two logically distinct phases, which we label phase 1 and phase 2, and which are necessary to describe the functioning of the decentralized trade mechanism taking place in the labor market. In phase 1, each worker/household and each intermediate firm do not know the partner they will be matched with. They must hence form expectations, conditional on the information sets Ω_{it}^H (for the worker) and Ω_{it}^F (for the firm), in order to define their demand and supply decisions. In phase 2, the match occurs in each local (decentralized) labor market and trade can actually occur.

We indicate a match with $(j, ma_t(j))$, where $ma_t(j) = i$ is a function determining the identity of agent i to be matched with firm j. The outcome of $ma_t(j)$ is ruled by a probability distribution which is common knowledge, i.i.d. and independent from the other stochastic variables of the economy. When, in phase 2, the values $(j, ma_t(j))$ are known, a competitive trade between the two partners occurs and in each local labor market the equilibrium values of $W_{(ji)t}$ and $N_{(ij)t}$ are determined. Given these values, the quantity Y_{jt} is set, for every matched couple (i, j), according to (10) and consequently the quantity Y_t is set according to (6). Given equilibrium in the final good market, Y_t is equal to the total demand for consumption $C_t = \int_0^1 C_i di$. Furthermore, given the individual agent i's budget constraint $C_{it} = \frac{l_{it-1}X_t}{P_t} = \frac{M_{it}}{P_t}$ and given equilibrium in the money market, C_t must be a function of the aggregate money supply and of P_t: this implies that also the price of final consumption is determined.

As for the role played by heterogeneity, notice that the intermediate firms are homogeneous (they all share the same production function, which is common knowledge). Hence each worker/household i knows, in phase 1, what type of j s/he will meet with. In other words, the type j in $(j, ma_t(j))$ is included in the information set Ω_{it}^H. Household i also knows that, according to (10), the wage W_{jt} depends on the aggregate production/demand level Y_t and on the consumption price P_t; these quantities can however be observed with certainty by each agent i, as the final good market operates in a centralized, Walrasian-type, fashion where information can flow without hindrance. Hence, when formulating his/her supply of labor, agent i does not need to solve a signal extraction problem: s/he only needs to compute expectations based on (common knowledge) public information (and on the knowledge of his/her type β_i). This in turn implies that each i also knows the actual value of the wage $W_{(ij)t}$ s/he will obtain in the local labor market.

Things are different for the intermediate firms. On the one side, as workers/households are heterogeneous and given decentralization, in phase 1 the intermediate firm j does not know with certainty the type of agent (i.e., β_i) it will match with. On the other side, firm j must also sell its output to the wholesale/aggregator, whose demand function (7) depends also on P_t and Y_t. These quantities are not observable by firm j; it only knows that in equilibrium $Y_t = C_t$, i.e., that aggregate production, and hence the demand for intermediate goods (7), depends on the level of the consumers' aggregate demand.

This imperfection in information and communication creates room for conjectures by intermediate firms on the level of aggregate demand. Analogously to Benhabib et al. (2015), we model this situation by assuming that in phase 1 each intermediate firm j receives a noisy signal S_{jt}, which contains stochastic information on the state of the local labor market and on the aggregate level of demand/production C_t, or Y_t. Hence, S_{jt} generally includes also a random variable representing the state of consumers' confidence – or *market sentiment* – conjectured by the intermediate firm(s) and related to the aggregate level of activity.

The structure of information available to firm j can then be summarized in the following way. The information set is $\Omega^F_{jt} = \{S_{jt}, \mathcal{I}_{t-1}, \mathcal{I}_{t-2}, ...\}$, where the $\mathcal{I}$s are the equilibrium levels of the economy's endogenous variables in the previous periods and the noisy signal S_{jt} is:

$$\ln S_{jt} = s_{jt} = \lambda b_i + (1-\lambda)\, z_t \tag{11}$$

where $b_i = \ln B_i$ is a function of the type β_i of household/worker (to be matched with), which will be described below. We assume that the firm only knows the probability distribution of b_i (and B_i), , which are the idiosyncratic components of the signal perceived by firm j in (11). The term $\ln Z_t = z_t$ is an i.i.d. random variable which describes the state of consumers' confidence conjectured by the intermediate producers, i.e., the market sentiment. As the j firms do not observe directly, at time t, the final good's market, they may think that different levels of aggregate consumption can be compatible with the distribution of consumers' preferences: there may be situations, or "times", of high or low aggregate demand. The intermediate producers only know the probability distribution of Z_t. In order to facilitate the existence of a tractable closed-form solution to the model, we assume that both Z_t and B_i are log-normally distributed with zero mean and non negative variance:

$$\begin{aligned} \ln B_i &= b_i \sim \mathbf{N}\left(0; \sigma_b^2\right) \\ \ln Z_t &= z_t \sim \mathbf{N}\left(0; \sigma_z^2\right) \end{aligned}$$

where $\sigma_b \geq 0$ and $\sigma_z^2 \geq 0$.

This description is again reminiscent of Lucas (1972) islands' model, as the individual firms receive a noisy signal from both an "idiosyncratic" (in the sense of sector-specific) source and an aggregate source. But whereas the element b_i is related to an intrinsic form of uncertainty, z_t is an extrinsic source of uncertainty: it represents the shared idea among intermediate firms of the possible aggregate consumers' willingness to buy. It is important to notice that the variance σ_z^2 will be treated as an endogenous variable to be determined within the model, consistently with the solution procedure described in the subsequent sections. Finally, $\lambda \geq 0$ represents a weighting coefficient, measuring the relevance attached by firm j to the idiosyncratic (or intrinsic) factors in the signal S_{jt}, which we assume to be common to all firms.[12]

[12] Benhabib et al. (2015) show that in this class of models λ can also be endogenized on the basis of a

In solving their choice problems, agents (intermediate firms and households) must form rational expectations based on the information available to them. Agents must hence formulate conjectures on the form of the possible equilibrium functions relating endogenous variables to parameters and exogenous variables. The two crucial endogenous variables describing the economy's aggregate equilibrium are Y_t and P_t, as the equilibrium conditions on the final good market and on the money market are:

$$Y_t = C_t; \qquad Y_t = \frac{M_t}{P_t} \tag{12}$$

We solve the model by employing the method of undetermined coefficients: we assume a particular set of conjectures on the equilibrium functions and then show that these conjectures are confirmed as viable equilibrium solutions. In particular, the solution conjectures on which we focus are:

$$\begin{aligned} Y_t &= \bar{Y} Z_t \\ P_t &= \bar{P} X_t^{\phi_1} M_{t-1}^{\phi_2} Z_t^{\phi_P} \end{aligned} \tag{13}$$

where $\bar{Y}$ and $\bar{P}$ are unknown (positive) constants and ϕ_1, ϕ_2 and ϕ_P are unknown coefficients. Our main goal is to show that the equilibrium output Y_t depends on the sentiment variable Z_t, so that extrinsic, sentiment-type fluctuations generated by Z_t can play a role in the equilibrium business cycle. Furthermore, we wish to show how heterogeneity, here encapsulated in β_i and b_i, affects the equilibrium values and their fluctuations.

By inserting the conjectures (13), together with the aggregate money supply equation $M_t = X_t M_{t-1}$, into the monetary equilibrium $Y_t = M_t/P_t$, we obtain:

$$\bar{Y} Z_t = \bar{P}^{-1} X_t^{1-\phi_1} M_{t-1}^{1-\phi_2} Z_t^{-\phi_P}$$

By matching the unknown constants and coefficients of both sides of the equation, we then derive:

$$\bar{y} = -\bar{p}; \qquad 1 = \phi_2; \quad 1 = \phi_1; \quad -1 = \phi_P \tag{14}$$

where lower-case letters indicate the logarithms of original quantities. The money neutrality - in the classical sense - of the resulting equilibrium output Y_t is incorporated into the conjecture $Y_t = \bar{Y} Z_t$; this appears to be consistent with the model's assumptions, as nominal prices are fully flexible.[13]

3.1 Households/workers choices: consumption and labor supply

Conjectures (13) allow us to complete the derivation of the household's decision rules. The equilibrium condition in the local labor market implies that the notation can be simplified

detailed description of the information gathering system and of its imperfections. As this would complicate the model without changing the substance of our story, we however prefer to retain the simpler approach of Benhabib (2012 and 2013) and take λ as exogenous.

[13] It can be shown that equilibria with money non-neutrality are impossble in this model. For example, by adopting for the aggregate output the conjecture $Y_t = \bar{Y} X_t^{\phi_3} M_{t-1}^{\phi_4} Z_t$, the resulting coefficients would always be: $\phi_3 = \phi_4 = 0$ at equilibrium. The proof is available from the authors upon request.

by writing the wage of worker i matched with firm j as W_{jt}. This wage is known by agent i, so that equation (5) can be solved with respect to W_{jt} yielding:

$$\begin{aligned} W_{,jt} &= N_{it}^{\psi} h_i^{-1} \text{ where:} \\ h_i &= \beta_i^{-1} E\left[\frac{X_{t+1}}{P_{t+1}} C_{it+1}^{-\gamma} \left|\Omega_{it}^H\right.\right] \\ &\quad + \beta_i E\left(\frac{C_{it+1}^{-\gamma}}{P_{t+1}} \begin{cases} \left(1-X_{t+1}^{-1}\right)^{1-\gamma} & \text{for } X_{t+1}\geq 1 \\ -\eta\left[-\left(1-X_{t+1}^{-1}\right)\right]^{1-\gamma} & \text{for } X_{t+1}<1 \end{cases} \left|\Omega_{it}^H\right.\right) \end{aligned} \tag{15}$$

Equation (8) together with the production function (9) implies:

$$C_{it+1} = \frac{X_{t+1}}{P_{t+1}} \left(P_t Y_t^{\frac{1}{\theta}} N_{j,t}^{1-\frac{1}{\theta}}\right) \tag{16}$$

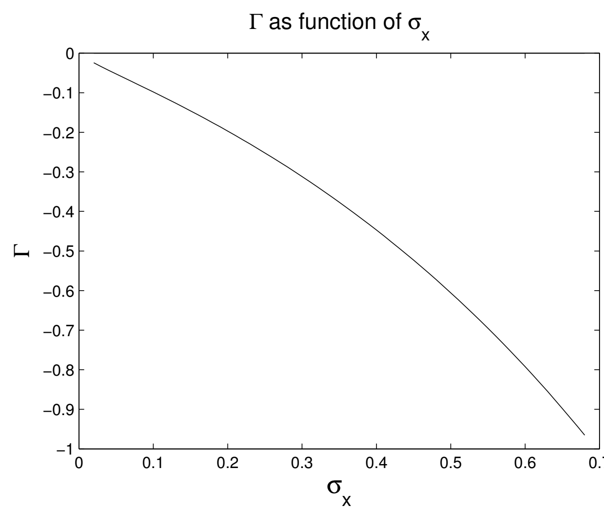
We make use of this equation, the monetary equilibrium (14) and the conjectures (13) in the expression of h_i to obtain:

$$h_i = N_{j,t}^{-\gamma\left(\frac{\theta-1}{\theta}\right)} \bar{P}^{-(1-\gamma)} M_{t-1}^{-(1-\gamma)} \left(P_t Y_t^{\frac{1}{\theta}}\right)^{-\gamma} E_{it}\left[X_t^{-(1-\gamma)} Z_{t+1}^{(1-\gamma)}\right] \left(\beta_i^{-1} + \Gamma\beta_i\right) \tag{17}$$

where:

$$\Gamma = \int_1^{\infty} X_{t+1}^{-1} \left(X_{t+1}-1\right)^{1-\gamma} f_X dX_{t+1} + \int_0^1 -\eta\left[X_{t+1}^{-1}\left[-\left(X_{t+1}-1\right)\right]^{1-\gamma}\right] f_X dX_{t+1} \tag{18}$$

as f_X is the known log-normal distribution of X_t, Γ is a definite integral that can actually be computed, given numerical values for γ, σ_x^2 and η. It can be shown numerically that, for a wide range of economically reasonable values for γ and η, the resulting Γ is negative and that there exists a negative relationship between the variance of the money supply σ_x^2 and the same PT component Γ. For instance, choosing $\eta = 2.25$, in line with a wide amount of experimental evidence (see, e.g., Tversky and Kahneman 1992), and $1-\gamma = 0.88$ for the curvature of consumption utility, a value which is also commonly accepted from experimental evidence in behavioral economics (Tversky and Kahneman 1992), the resulting values of Γ as a function of σ_x are shown in figure 1.



The explanation of this behavior is in line with that provided in Ciccarone and Marchetti (2013). When formulating their forecasts on the economy's evolution, households/workers consider the possibility that monetary shocks induce gains or losses with respect to their reference wealth. As their rational expectations will be (on average) fulfilled, they take into account this effect when formulating a lower average supply of labor (and/or demand of consumption) as a form of precautionary behavior. If the volatility of the returns on their money holding (σ_x) increases, agents tend to sharpen this precautionary behavior, due to the increased possibility of losses and to the fact that for them losses loom larger than gains. This will in turn drive an increase in the absolute value of Γ, which implies a reduction of h_i in equation (15) and hence, *coeteris paribus*, a reduction of the labor supplied by the individual worker. We hence take $\Gamma < 0$ for the subsequent analysis.

Given the monetary equilibrium $Y_t = M_t/P_t$ and the structure of information, agent i observes the value of X_t, so that $E_{it}\left(X_t^{-(1-\gamma)}\right) = X_t^{-(1-\gamma)}$. His/her expectation of the sentiment variable Z_t is simply the unconditional average, as s/he does not need to operate any signal extraction: $E_{it}\left(Z_{t+1}^{(1-\gamma)}\right) = e^{\frac{1}{2}(1-\gamma)^2\sigma_z^2}$. From equation (17) we then obtain:

$$h_i = \bar{P}^{-(1-\gamma)} M_{t-1}^{-(1-\gamma)} X_t^{-(1-\gamma)} e^{\frac{1}{2}(1-\gamma)^2\sigma_z^2} \left(P_t Y_t^{\frac{1}{\theta}} N_{j,t}^{1-\frac{1}{\theta}}\right)^{-\gamma} \left(\beta_i^{-1} + \beta_i \Gamma\right)$$

and the worker's labor supply equation $W_{,jt} = N_{it}^{\psi} h_i^{-1}$ can be rewritten as:

$$\begin{aligned} W_{,jt} &= N_{jt}^{\psi+\gamma\left(\frac{\theta-1}{\theta}\right)} H_i^{-1} \quad \text{where:} \\ H_i &= \bar{P}^{-(1-\gamma)} M_{t-1}^{-(1-\gamma)} X_t^{-(1-\gamma)} e^{\frac{1}{2}(1-\gamma)^2\sigma_z^2} \left(P_t Y_t^{\frac{1}{\theta}}\right)^{-\gamma} B_i; \\ B_i &= \beta_i^{-1} + \beta_i \Gamma \end{aligned} \tag{19}$$

Given the assumption that the B_i are log-normally distributed with zero mean and variance $\sigma_b \geq 0$**,** the probability distribution function of the β_i can be recovered.[14] For the time being it is however sufficient to note that given that assumption and the fact that $\Gamma < 0$, the β_i are subject to belong to the set $\beta_i \in \left(0; \beta_{\max} = \sqrt{-\Gamma^{-1}}\right)$. In (19), the term B_i measures the worker's propensity to supply labor, as affected by the behavioral elements Γ and β_i. When $\beta_i \to 0$, the term $B_i \to +\infty$ and the worker's supply is infinitely elastic for a given level of the wage; this is due to the fact that the framing effect in utility (3) disappears and at the same time the utility of consumption becomes arbitrarily high. When $\beta_i = \sqrt{-\Gamma^{-1}}$, it is $B_i = 0$ and the worker supplies no labor: in this case a high framing effect magnifies the PT component in (3) and makes the worker extremely cautious, to the point of renouncing his/her labor income.

3.2 Intermediate firms: signal extraction in the local labor market

In the local labor market equilibrium demand and supply equate, $N_{it} = N_{jt}$, and the intermediate firm j knows that the prevailing wage will be coherent with equation (19). By substituting this expression of W_{jt} into the firm's optimality condition (10) we obtain:

$$N_{j=i,t} = \left(1 - \frac{1}{\theta}\right)^{\theta} E\left(N_{i=jt}^{\psi+\gamma\left(\frac{\theta-1}{\theta}\right)} H_i^{-1} \middle| \Omega_{jt}^F\right)^{-\theta} E\left(P_t Y_t^{\frac{1}{\theta}} \middle| \Omega_{jt}^F\right)^{\theta} \tag{20}$$

The firm faces uncertainty over B_i (the type of agent i) and treats b_i and z_t as random variables. In computing the rational expectations in (20), it must solve a signal extraction problem: it observes signal S_{jt} given by (11), which is the relevant element in the information set Ω_{jt}^F, and adopts the solution conjectures (13). This allows us to write equation (20) as:

$$N_{j,t}^{1+\theta\psi+\gamma(\theta-1)} = \bar{Y}^{1-\gamma} e^{\frac{\theta}{2}(1-\gamma)^2\sigma_z^2} \left(\frac{\theta-1}{\theta}\right)^{\theta} E\left(X_t Z_t^{\left(\frac{1}{\theta}-1\right)\gamma} B_i^{-1} \middle| S_{jt}\right)^{-\theta} E\left(X_t Z_t^{\frac{1}{\theta}-1} \middle| S_{jt}\right)^{\theta} \tag{21}$$

As x_t, z_t and b_i are all normal and uncorrelated, the conditional expectations in (21) can be computed:

$$E\left(X_t Z_t^{\frac{1}{\theta}-1} \middle| S_{jt}\right) = \exp\left\{\frac{(1-\lambda)\left(\frac{1}{\theta}-1\right)\sigma_z^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}\left[\lambda b_i + (1-\lambda) z_t\right] + \frac{1}{2}Q_Y\right\} \tag{22}$$

$$E\left(X_t Z_t^{\left(\frac{1}{\theta}-1\right)\gamma} B_i^{-1} \middle| S_{jt}\right) = \exp\left\{\frac{\left(\frac{1}{\theta}-1\right)\gamma(1-\lambda)\sigma_z^2 - \lambda\sigma_b^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}\left[\lambda b_i + (1-\lambda) z_t\right] + \frac{1}{2}Q_B\right\}$$

where the Q's are functions of the model's parameters only:

$$Q_Y = \sigma_x^2 + \left(\frac{1}{\theta}-1\right)^2\sigma_z^2 - \frac{\left[(1-\lambda)\left(\frac{1}{\theta}-1\right)\sigma_z^2\right]^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2} \tag{23}$$

$$Q_B = \sigma_x^2 + \left(\frac{1}{\theta}-1\right)^2\gamma^2\sigma_z^2 + \sigma_b^2 - \frac{\left[\left(\frac{1}{\theta}-1\right)\gamma(1-\lambda)\sigma_z^2 - \lambda\sigma_b^2\right]^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}$$

[14] See section 5.

By substituting the expression (22) back into (21) and by making use of the production function (9), the equilibrium output of firm j turns out to be:

$$Y_{jt} = \bar{N} \exp \left\{ \theta \frac{\lambda \sigma_b^2 + (1-\gamma) \left(\frac{1}{\theta} - 1\right) (1-\lambda) \sigma_z^2}{[1 + \gamma (\theta - 1) + \theta \psi] \left[\lambda^2 \sigma_b^2 + (1-\lambda)^2 \sigma_z^2\right]} (\lambda b_i + (1-\lambda) z_t) \right\} \tag{24}$$

where $\bar{N}$ includes only the model's parameters and the unknown stationary output $\bar{Y}$:

$$\begin{aligned} \bar{N} &= K \bar{Y}^{\frac{1-\gamma}{1+\gamma(\theta-1)+\theta\psi}} \exp \left(\frac{1}{1 + \gamma (\theta - 1) + \theta \psi} \right) \left(\frac{\theta}{2} Q_Y - \frac{\theta}{2} Q_B \right) \\ K &= \left(1 - \frac{1}{\theta}\right)^{\frac{\theta}{1+\gamma(\theta-1)+\theta\psi}} \end{aligned}$$

4 Macroeconomic Equilibrium and sentiment-driven fluctuations

In order to verify that the original conjectures on the equilibrium solutions were correct, overall output must be computed. By inserting (24) into the aggregate production function (6), we obtain:

$$Y_t = \left[\int \left(\bar{N} \exp \left\{ \left(\frac{\theta [\lambda b_i + (1-\lambda) z_t]}{1 + \gamma (\theta - 1) + \theta \psi} \right) \frac{\lambda \sigma_b^2 + (1-\lambda)(1-\gamma) \left(\frac{1}{\theta} - 1\right) \sigma_z^2}{\lambda^2 \sigma_b^2 + (1-\lambda)^2 \sigma_z^2} \right\} \right)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{25}$$

where the only indexed elements are the b_i and all the other elements can be factored out of the integral. As matching in the decentralized labor market is pair-wise, the integration can be indifferently carried out over dj or over di and, since the b_i are normally distributed, the law of great numbers allows us to approximate the integral with the average value to obtain:

$$\begin{aligned} Y_t &= \bar{V} \exp \left\{ \left(\frac{\theta \left[\lambda \sigma_b^2 + (1-\lambda)(1-\gamma) \left(\frac{1}{\theta} - 1\right) \sigma_z^2\right] (1-\lambda)}{[1 + \gamma (\theta - 1) + \theta \psi] \left[\lambda^2 \sigma_b^2 + (1-\lambda)^2 \sigma_z^2\right]} \right) z_t \right\} \\ \bar{V} &= K \bar{Y}^{\frac{1-\gamma}{1+\gamma(\theta-1)+\theta\psi}} \exp \left(\frac{\theta (Q_Y - Q_B + Q_V)}{2 [1 + \gamma (\theta - 1) + \theta \psi]} \right) \\ Q_V &= \frac{(\theta - 1) \lambda^2 \sigma_b^2}{1 + \gamma (\theta - 1) + \theta \psi} \left(\frac{\lambda \sigma_b^2 + (1-\gamma) \left(\frac{1}{\theta} - 1\right) (1-\lambda) \sigma_z^2}{\lambda^2 \sigma_b^2 + (1-\lambda)^2 \sigma_z^2} \right)^2 \end{aligned} \tag{26}$$

Now compare the log of expression (26) with the log of the original conjecture, $y_t = \bar{y} + z_t$, to write:

$$\bar{y} + z_t = \bar{v} + \left(\frac{\theta}{1 + \gamma (\theta - 1) + \theta \psi} \right) \frac{\lambda \sigma_b^2 + (1-\lambda)(1-\gamma) \left(\frac{1}{\theta} - 1\right) \sigma_z^2}{\lambda^2 \sigma_b^2 + (1-\lambda)^2 \sigma_z^2} (1-\lambda) z_t$$

For the conjecture to be coherent with the resulting equilibrium, two equations must hence be satisfied:

$$\bar{y} = \bar{v} \quad \text{and:} \quad \left(\frac{\theta (1-\lambda)}{1+\gamma(\theta-1)+\theta\psi} \right) \frac{\lambda\sigma_b^2 + (1-\lambda)(1-\gamma)\left(\frac{1}{\theta}-1\right)\sigma_z^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2} = 1$$

Being $\bar{V} > 0$, $\bar{y} = \bar{v}$ is a viable relation and the value of $\bar{Y}$ can be determined by making use of the expressions in (26). The second equation poses instead some limitations on the values that can be taken up by the model's parameters. To see this, rewrite it as:

$$\sigma_z^2 = \left[\frac{\theta(1-\lambda) - [1+\gamma(\theta-1)+\theta\psi]\lambda}{\theta(1+\psi)(1-\lambda)^2} \right] \lambda\sigma_b^2 \tag{27}$$

As the variances σ_b^2 and σ_z^2 must be non negative, for (27) to hold the following inequality must be satisfied:

$$\lambda < \bar{\lambda} = \frac{\theta}{1+\theta+\theta\psi+\gamma(\theta-1)} < 1 \tag{28}$$

This value of λ is included in its definition interval $(0;1)$ and is hence a viable value.

The conclusion is that when:

$$\lambda \in \left(0; \bar{\lambda} = \frac{1}{2+\psi+\varepsilon}\right); \qquad \varepsilon = (1-\gamma)\left(\frac{1}{\theta}-1\right) \in (-1;\ 0) \tag{29}$$

a self-fulfilling, sentiment equilibrium for this economy exists, with the equilibrium output provided by equations (26). In other terms, any value of λ satisfying (29), makes the conjectures (13), in particular $y_t = \bar{y} + z_t$, the actual equilibrium solutions of the model and the variance σ_z^2 of the sentiment variable z_t is the one set by equation (27). In equilibrium, output volatility σ_y^2 equals σ_z^2 and, given equation (27), it is related to the parameters of the population distribution of β_i via the relation $B_i = \left(\beta_i^{-1} + \beta_i\Gamma\right) \sim \ln \mathbf{N}\left(0, \sigma_b^2\right)$.

We may hence say that the actual value of λ determines the nature of macroeconomic equilibrium. If $\lambda = 0$, the signal does not include any kind of information on the worker's type, so that the signal extraction problem disappears and sentiments can play no role in generating equilibrium fluctuations of aggregate output. In this situation the unique possible equilibrium compatible with rational expectations requires the solution conjecture $y_t = \bar{y}$, that is, output must be stationary and equal to a "fundamental" value $\bar{y}$. If $\lambda \geq \bar{\lambda}$, it can be shown[15] that if agents make the conjecture $y_t = \bar{y} + z_t$, the absolute value of the difference between the actual output and the conjectured one will always be strictly positive, so that the same conjecture is incompatible with rational expectations. When $\lambda \geq \bar{\lambda}$, it is $\frac{\theta(1-\lambda)}{1+\gamma(\theta-1)+\theta\psi}\left[\frac{\lambda\sigma_b^2+(1-\gamma)\left(\frac{1-\theta}{\theta}\right)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_b^2+(1-\lambda)^2\sigma_z^2}\right] < 1$ and this implies that (in absolute value) equilibrium output from (26) is always lower than the conjectured one (in absolute value).[16] This suggests that when $\lambda \geq \bar{\lambda}$ the unique possible equilibrium, under rational expectations, is the fundamental one: $y_t = \bar{y}$.

[15] In a way similar to that discussed by Bhenhabib et al. (2015, p. 560).

[16] See Appendix 3.

Finally, we can calculate the equilibrium value of stationary output by matching the coefficients $\bar{Y} = \bar{V}$ and solving with respect to $\bar{Y}$ the equation:

$$\bar{Y} = K\bar{Y}^{\frac{1-\gamma}{1+\gamma(\theta-1)+\theta\psi}} \exp\left(\frac{\theta\left[Q_Y - Q_B + Q_V + (1-\gamma)^2\sigma_z^2\right]}{2\left[1+\gamma(\theta-1)+\theta\psi\right]}\right)$$

The solution, expressed in logs, is:

$$\begin{aligned} \bar{y} &= \bar{k} + \frac{1}{2(\gamma+\psi)}\left(Q_Y - Q_B + Q_V + (1-\gamma)^2\sigma_z^2\right); \qquad (30) \\ \bar{k} &= \frac{1+\gamma(\theta-1)+\theta\psi}{\theta(\gamma+\psi)}\ln K \end{aligned}$$

In analogy with Benhabib et al. (2012; 2013), the model can also be solved under a slightly more general assumption on the composition of the signal (11). It can in fact be shown that if the signal were equal to $s_{jt} = \lambda b_i + (1-\lambda) z_t + v_{jt}$, where $v_{jt} \sim \mathbf{N}\left(0;\ \sigma_v^2\right)$ represents a labor-market specific source of intrinsic uncertainty, the conjecture $y_t = \bar{y} + z_t$ would remain a solution of the model (see Appendix 2).

According to condition (29), the existence of a sentiment-driven equilibrium is affected by $\varepsilon = (1-\gamma)\left(\frac{1}{\theta}-1\right) < 0$. Due to the log-normal distribution of the model's stochastic variables and to the linear production function (9), equation (21) can also be written in this way:

$$\begin{aligned} y_{ji} &= a + \frac{\theta\left[E\left(x_t + \left(\frac{1}{\theta}-1\right)y_t \middle| s_{jt}\right) - E\left(x_t + \gamma\left(\frac{1}{\theta}-1\right)y_t - b_i \left|s_{jt}\right.\right)\right]}{1+\gamma(\theta-1)+\theta\psi} \\ &= a_0 + \frac{\varepsilon}{1+\psi+\varepsilon}E\left(y_t\left|s_{jt}\right.\right) - \frac{1}{1+\psi+\varepsilon}E\left(-b_i\left|s_{jt}\right.\right) \qquad (31) \end{aligned}$$

where a and a_0 depend on the model's parameters only.

Hence $\varepsilon \in (\gamma-1;\ 0)$ is the coefficient that rules the degree of strategic substitutability between the firms' decisions, i.e., the relationship between y_{jt} and y_t. As shown by Benhabib et al. (2015, pp. 561-63), this modeling framework can incorporate both strategic complementarity and substitutability in production, and in a general model of this type both cases can be compatible with sentiment-driven fluctuations in a rational expectations equilibrium. Substitutability or complementarity of the individual firm's decision rule depend on the specific assumptions of the model. In our scheme, the strategic substitutability displayed by (31) stems from the Dixit-Stiglitz form of the aggregate production function (6) coupled with the restriction $\gamma \in (0;1)$, which is required by PT. Given (29), the strategic substitutability here guaranteed by $\varepsilon < 0$ increases the possibility that a sentiment-driven equilibrium exists.

Furthermore, equation (31) shows that as the degree of monopoly power $1/\theta$ raises from 0 to 1 (θ decreases from ∞ to 1) ε increases from the lowest possible value $(\gamma-1)$ to 0, the degree of substitutability $\left|\frac{\varepsilon}{1+\psi+\varepsilon}\right|$ decreases towards zero and the threshold $\bar{\lambda} = (2+\psi+\varepsilon)^{-1}$ falls towards the lower bound $(2+\psi)^{-1}$: as monopoly power in the intermediate goods sector increases, the possibility of having a sentiment-driven equilibrium becomes dimmer. This occurs because a higher degree of competition increases the degree of strategic substitutability and favours the emergence of self-fulfilling equilibrium by exacerbating the coordination problem generated by the decentralized nature of trade taking place in the labor market. When the intermediate firm can act as a pure monopolist, it is instead more free to set its output independently from the signals stemming from aggregate demand sentiments. Nevertheless, even if $\frac{\varepsilon}{1+\psi+\varepsilon}E\left(y_t\,|s_{jt}\right)$ is almost zero, the term $E\left(-b_i\,|s_{jt}\right)$ does not vanish, as the intermediate firm is still unable to exactly pin down the worker's type b_i: a higher degree of monopoly power can weaken the possibility of a self-fulfilling equilibrium, but cannot eliminate it as the threshold $\bar{\lambda}$ cannot fall below $(2+\psi)^{-1}$.

Note that the emergence of a sentiment-driven equilibrium is also affected by the elasticity of the worker's labor supply, $\varepsilon_{LS}=\frac{1}{\psi}$. A higher elasticity (a smaller value of ψ) favours the emergence of a sentiment-driven equilibrium while a greater rigidity ($\psi\to\infty$) tends to eliminate it. Under a rigid labor supply, the amount N_{jt} is independent from the wage and from h_i, so that intermediate firms face no uncertainty relative to the type of worker they will be matched with and consequently the hindrances to information flow in the decentralized labor markets are removed, together with the source of a possible sentiment-driven equilibrium.

According to the equilibrium output solution $y_t=\bar{y}+z_t$ discussed in the previous section, the features of the distribution of the β_i play a role in determining the level of stationary output $\bar{y}$. From equation (30) and the definitions of Q_Y, Q_B and Q_V in (23) and (26), $\bar{y}$ can be written as:

$$\bar{y}=\bar{k}+\frac{\left[(\theta-1)(1+\psi)(1+\gamma)+\theta(1-\gamma)^2\right]\sigma_z^2+\theta\left[\frac{\lambda(\theta-\lambda)[1+\gamma(\theta-1)+\theta\psi]}{\theta^2(1-\lambda)^2}-1\right]\sigma_b^2}{2\theta(\gamma+\psi)}$$

Under a sentiment equilibrium, σ_z is proportional to σ_b, as shown by (27) and it would hence be possible to study the effects of an increase in the variance σ_b^2 on $\bar{y}$. However, given our focus on sentiment fluctuations, we choose to concentrate our attention on the impact of the distribution of the β_i on the self-fulfilling equilibrium implied in condition (29).[17]

5 The effects of framing heterogeneity on sentiment-driven fluctuations

The probability distribution function of β_i, f_β, can be obtained from the relationship $B_i=\frac{1}{\beta_i}-|\Gamma|\,\beta_i$, where $\beta_i\in\left(0;\;1/\sqrt{|\Gamma|}\right)$ and $\Gamma<0$, and from the assumption on the log-normality of the probability distribution function of B_i:

$$f_B=\frac{1}{B_i\sigma_b\sqrt{2\pi}}\exp\frac{1}{2}\left(-\frac{(\ln B_i)^2}{\sigma_b^2}\right)$$

[17]It is straightforward to show that the impact of σ_b on $\bar{y}$ is analogous to that obtained by Benhabib et al. (2012): the effect may be positive or negative depending on the values of the model's parameters. Computations are available upon request.

By applying the approximate rule:

$$f_\beta(\beta_i) = f_B(B(\beta_i)) \left| \frac{\partial B_i}{\partial \beta_i} \right|$$

we obtain:

$$f_\beta(\beta_i) = \frac{1}{\beta_i \sigma_b \sqrt{2\pi}} \left(\frac{1 + |\Gamma| \beta_i^2}{1 - |\Gamma| \beta_i^2} \right) \exp \frac{1}{2\sigma_b^2} \left[- \left(\ln \left(\frac{1 - |\Gamma| \beta_i^2}{\beta_i} \right) \right)^2 \right] \tag{32}$$

The probability distribution function (32) can be studied numerically by imposing specific values to the parameters γ, σ_x and η. For instance, we may choose for η and γ the values used in section 3.1, i.e., $\eta = 2.25$ and $\gamma = 0.22$; by setting $\sigma_x = 0.2$ and $\sigma_b = 0.31$ we then obtain the value $|\Gamma| = 0.197$. The resulting probability distribution function of β_i is depicted in Figure 2:

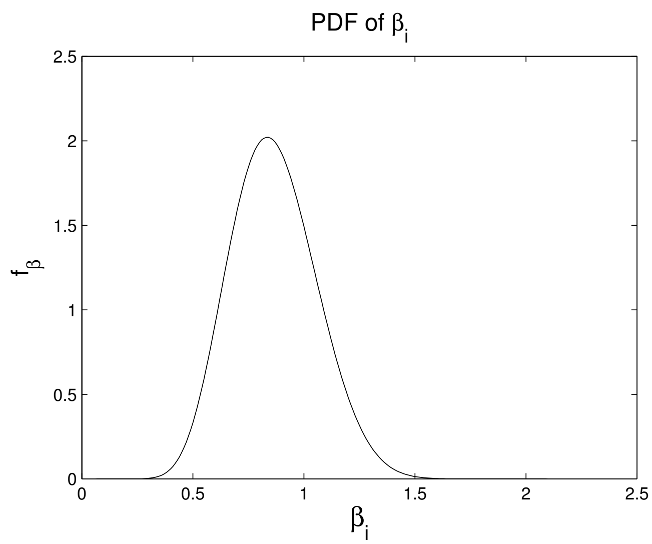


Figure 2: The distribution of the β_i as recovered from that of the B_i. The support of β_i is (0;2.294) under the parameterisation: $\eta = 2.25$; $\gamma = 0.22$; $\sigma_x = 0.2$; $\sigma_b = 0.31$.

The PDF is quasi-symmetric with a unique maximum. Numerical simulations show that if σ_b grows, the function becomes more asymmetric and (by keeping $\sigma_b < \sigma_x$) flatter; furthermore, an increase in $|\Gamma|$ shifts the function horizontally and changes the height of the figure, leaving however the general shape unchanged.

We can now exploit the function (32) to understand the relationship between σ_β and σ_b. A numerical computation of this relationship, for $|\Gamma| = 0.197$, is shown in figure 3:

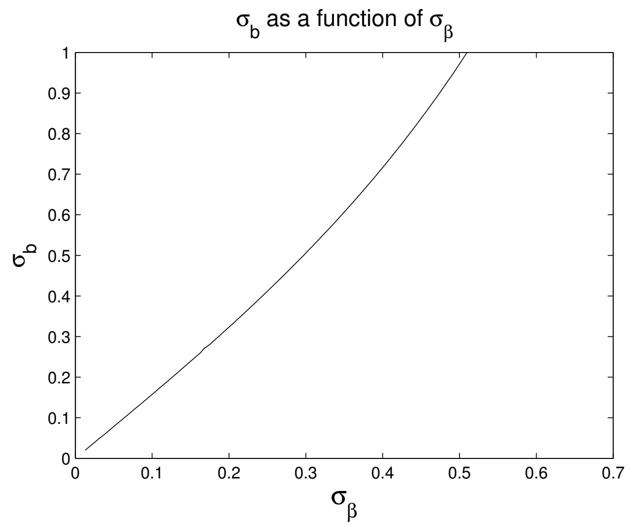


Figure 3: The graph depicted in the figure is obtained by assigning to σ_b values in the interval [0; 1]; then, for each value of σ_b, a PDF $f_\beta(\beta_i)$ is computed by setting $|\Gamma| = 0.197$ in equation (32) and the corresponding standard deviation σ_β is obtained. The parameterization required for $|\Gamma| = 0.197$ is the same as that of figure 2: $\eta = 2.25$; $\gamma = 0.22$; $\sigma_x = 0.2$; $\sigma_b = 0.31$.

This relationship is monotonic and increasing, and it is robust to variations in the value of $|\Gamma|$. While agents' deviations from standard rationality, as measured by parameters β_i, are not sufficient *per se* to generate sentiment-driven fluctuations, this shows that the more dispersed are these deviations, the greater is σ_β and the higher is the volatility of the economy due to sentiment fluctuations. A high dispersion increases in fact the variability of the intermediate firms' sentiments on the state of aggregate demand and in this way amplifies the volatility of (real) macroeconomic variables. Furthermore, as it is $\sigma_b = 0$ when $\sigma_\beta = 0$, homogeneity in the framing parameter prevents the emergence of a self-fulfilling equilibrium. This is coherent with the theoretical approach to sentiment fluctuations developed by Angeletos and La'O (2013): in order to obtain this type of fluctuations, agents' heterogeneity must be coupled with a form of trade decentralization, a mechanism which is included in the modelling framework of Benhabib et al. (2013; 2015) and in our model too.

A point of particular interest in the context of this model is the relationship between, on the one side, the volatility of the money market, synthesized by the variance σ_x^2, which affects the value of $|\Gamma|$ and, on the other side, the features of the distribution of β_i, which affects the possibility of a self-fulfilling equilibrium and, via equation (27), the overall volatility of the economy. In order to analyze this relationship, we carry out a numerical analysis of the sensitivity of σ_b and σ_β (which are reciprocally related through the probability distribution function (32)) to changes in $|\Gamma|$ produced by variations in the monetary variance σ_x^2.

First recall that the numerical simulation producing the results shown in figure 1 of section 3.1 implies that there exists an increasing relationship between σ_x and the absolute value $|\Gamma|$. Taking into account this relationship together with the PDF (32), we carry out a numerical analysis of the relationship between σ_x and σ_b, for a given level of β_i's standard deviation σ_β. The result is shown in figure 4.

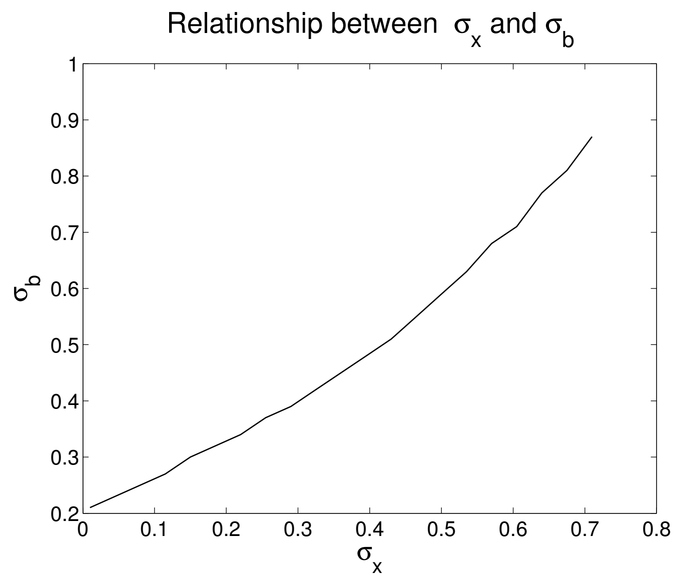


Figure 4: To obtain the graph depicted in the figure, start with the initial small value $\sigma_x = 0.01$ and compute the value of $|\Gamma|$; then insert these values into the approximate distribution (32) to obtain a relationship between σ_b and σ_β as that shown in figure 3. Repeat this procedure for increasing values of σ_x (and of $|\Gamma|$), so as to get each time two series of values of (i.e., a relationship between) σ_b and σ_β. Finally, draw the graph which obtains by selecting from each of these relationship the value of σ_b corresponding to $\sigma_\beta = 0.22$ and the associated value of σ_x.

Figure 4 shows that a higher value of $|\Gamma|$, generated by greater money market instability σ_x, not only implies a reduction of h_i in equation (15) and hence, *coeteris paribus*, a reduction of the labor supplied by the individual worker, but is also associated with a greater dispersion of the b_i component of the signal s_{jt}. This produces a greater variability in market sentiments and amplifies the sentiment-driven volatility of real macroeconomic variables. The economic intuition underlying this result is that more volatile money/financial markets, by increasing the value of the PT component, not only induce loss averse agents to be more cautious in providing time and effort in exchange for income to be invested with the prospect of possibly higher losses, but also increase the noise of the signal intermediate firms receive and upon which they form their labor demand schedules.

At a first sight, as far as increases in σ_x can be thought to be generated by policy decisions alone, these results appear to be consistent with a traditional view of the conduct of monetary policy, i.e., in the direction of conservative monetary policies. However, in this economy σ_x may also encapsulate sources of uncertainty and shocks which are different from the decisions of the monetary authorities and are related to structural factors, or features of the money market. When these external factors are particularly strong and bring about a significant increase in σ_x, monetary policy should actively counteract them, so as to reduce the overall value of σ_x as much as possible.

6 Conclusions

The model presented in this paper identifies a novel channel through which the volatility of the monetary/financial sector can amplify the instability of the real macroeconomic variables produced by self-fulfilling market sentiments. This mechanism is based on the simultaneous existence of a coordination problem generated by a pair-wise matching process in decentralized labor markets and heterogeneity in households' preferences affected by loss aversion and narrow framing.

In order to tell our story in an analytically tractable way, we have built an overlapping generations economy with a continuum of heterogeneous households, each one endowed with a specific utility function, and a continuum of markets for labor inputs, where a decentralized pair-wise process randomly matches a worker and a firm. The households, who suffer from narrow framing and loss aversion, evaluate changes in their financial wealth *per se*, in addition to their effects on future consumption. Households' heterogeneous preferences prevent firms from knowing in advance the type of household/worker they will be matched with. In order to determine their demands for labor they hence compute rational expectations conditional on the information provided by a noisy signal they receive on the partner's type and on the aggregate level of demand (on the market sentiment). The simultaneous presence of imperfect information and of households' heterogeneity hence imposes firms to solve this signal extraction problem and leaves room for conjectures on the level of aggregate demand to play a role.

Given this set up, we have demonstrated that in this economy, for a wide range of parameters' values, there exists a self-fulfilling, sentiment-driven equilibrium, whose emergence is facilitated by: i) a high wage elasticity of the labor supply, which increases firms' uncertainty on the type of worker they will be matched with; ii) the strategic substitutability of firms' decisions, in its turn fostered by a high degree of competition in goods markets, which exacerbate the coordination problem generated by the decentralized nature of trade occurring in the labor markets. We have also demonstrated that even though heterogeneity in the households' framing attitude is not sufficient *per se* to generate sentiment-driven fluctuations, the more dispersed are these deviations the higher is the volatility of the economy due to sentiment fluctuations. A higher dispersion magnifies in fact the variability of the intermediate firms' sentiments on the state of aggregate demand and in this way amplifies the volatility of (real) macroeconomic variables.

Our conclusion is that, given the above results, deviations from standard rationality affecting households' consumption-saving choices can create a new theoretical mechanism favoring the transmission of volatility from monetary/financial markets to real macroeconomic variables. This conclusion can be reached in two steps. First, the volatility of monetary/financial markets affects the choices under risk of consumers/savers characterized by loss aversion and a narrow framing attitude; when this volatility increases, the effect of loss aversion becomes higher, as the possibility of experiencing losses increases, and agents become more cautious in supplying labor. Second, in the presence of heterogeneous narrow framing attitudes, this increased value of the loss aversion component makes the signal upon which intermediate firms form their labor demand schedules noisier; this, in its turn, allows for a greater variability in market sentiments and hence in real economic activity. This channel of transmission of monetary (or financial) volatility to real macroeconomic variables is independent from the presence of nominal rigidities in price or wage setting, which in our scheme are set in every time period. It is worth stressing that this conclusion does not necessarily support a predictable and stable conduct of monetary policy, as the volatility of monetary/financial markets may be due to sources of uncertainty and shocks related to structural factors, or features of the money market, which monetary policy should actively counteract.

When applied to the sudden interruption of the Great Moderation produced by the burst of financial instability in 2007-2008,[18] our model suggests a new channel through which it may have affected macroeconomic volatility. Increased financial instability favours greater dispersion in heterogenous loss averse households' choices, this increases the uncertainty faced by the firms making hiring and production decisions, and in this way it generates greater variability in market sentiments and wider business cycle fluctuations.

APPENDIX

1. We show that under the conjecture $Y_t = \bar{Y}$ the weight λ must be equal to one. First notice that adopting this conjecture the coefficient H_i in equation (19) is now equal to:

$$H_i = \bar{P}^{-(1-\gamma)} M_{t-1}^{-(1-\gamma)} X_t^{-(1-\gamma)} \left(P_t Y_t^{\frac{1}{\theta}} \right)^{-\gamma} B_i$$

By adopting the same procedure of the main text, the equilibrium quantity in the $i-j$ labor market, as summarized by equation (21), now becomes:

$$N_{j,t}^{1+\theta\psi+\gamma(\theta-1)} = \bar{Y}^{1-\gamma} \left(\frac{\theta-1}{\theta} \right)^{\theta} E\left(X_t B_i^{-1} \left| S_{jt} \right. \right)^{-\theta} E\left(X_t \right| S_{jt})^{\theta} \tag{33}$$

The stochastic shock X_t is independent from every other variable and hence the two expectations $E(X_t)$ in the former equation simplify out; it then remains to compute only the conditional expectation $E\left(B_i^{-1} \left| S_{jt} \right.\right)$. Given the log-normal distribution of B_i and

[18] Even though there exist several indexes of financial conditions (see, e.g., Illing and Liu, 2006; Nelson and Perli, 2007; Hakkio and Keeton, 2009; Hatzius et al., 2010), it is by now agreed that financial instability increased after 2007, even though the financial system may have healed after mid-2009 (Brave and Butters, 2011).

$S_{jt} = B_i^{\lambda} Z_t^{1-\lambda}$, the signal extraction problem can be solved and the result is:

$$E\left(B_i^{-1}/S_{jt}\right) = \exp\left\{-\frac{\lambda\sigma_b^2}{\lambda^2\sigma_b^2 + (1-\lambda)\sigma_z^2}\left[\lambda b_i + (1-\lambda) z_t\right] + \frac{1}{2}Q_F\right\}$$

$$Q_F = \left(\frac{(1-\lambda)^2\sigma_z^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}\right)\sigma_b^2$$

By substituting this solution into (33) and by making use of the production function $Y_{jt} = N_{jt}$, the following value of the jth equilibrium output can be obtained:

$$Y_{jt} = \hat{N}\exp\left\{\frac{\theta\lambda\sigma_b^2\left[\lambda b_i + (1-\lambda) z_t\right]}{\left[1 + \gamma(\theta - 1) + \theta\psi\right]\left[\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2\right]}\right\} \tag{34}$$

where $\hat{N}$ is a positive constant. Now equation (34) can be used to compute the equilibrium level of aggregate output following the procedure adopted in section 4; the result is:

$$Y = \hat{V}\exp\frac{\theta}{\left[1 + \gamma(\theta - 1) + \theta\psi\right]}\left(\frac{\lambda\sigma_b^2(1-\lambda)}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}z_t\right) \tag{35}$$

where $\hat{V}$ is a positive constant. Taking the logs of equation (35) and matching the coefficient of the original conjecture $y_t = \bar{y}$, we obtain this equation:

$$\bar{y} = \hat{v} + \frac{\theta}{\left[1 + \gamma(\theta - 1) + \theta\psi\right]}\left(\frac{\lambda\sigma_b^2(1-\lambda)}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2}\right)z_t$$

Hence the coefficient multiplying z_t must be equal to zero and it is straightforward to see that this can only happen when $\lambda = 1$ or $\lambda = 0$.

2. The model can be analyzed under a slightly more general assumption on the composition of the signal (11), based on the possibility for s_{jt} to be also affected by a labor-market specific source of intrinsic uncertainty, represented by an uncorrelated i.i.d. random variable $v_{jt} \sim \mathbf{N}\left(0;\ \sigma_v^2\right)$:

$$s_{jt} = \lambda b_i + (1-\lambda) z_t + v_{jt} \tag{36}$$

By carrying out the solution procedure employed in the main text, we find that under the conjecture $Y_t = \bar{Y}Z_t$ the same equation (21) crops up, so that we need to compute the expectations $E\left(Z_t^{\left(\frac{1}{\theta}-1\right)\gamma}B_i^{-1}\middle| S_{jt}\right)$ and $E\left(Z_t^{\frac{1}{\theta}-1}\middle| S_{jt}\right)$ with the new definition (36). Under the assumed probability distributions for b_i, z and v_j we obtain:

$$E\left(Z_t^{\frac{1}{\theta}-1}\middle| S_{jt}\right) = \exp\left\{\frac{(1-\lambda)\left(\frac{1}{\theta}-1\right)\left[\lambda b_i + (1-\lambda) z_t + v_{jt}\right]\sigma_z^2}{\lambda^2\sigma_b^2 + (1-\lambda)^2\sigma_z^2 + \sigma_v^2} + \frac{1}{2}Q_I\right\}$$

$$E\left(Z_t^{\left(\frac{1}{\theta}-1\right)\gamma}B_i^{-1}\left|S_{jt}\right.\right)=\exp\left\{\frac{\left[\left(\frac{1}{\theta}-1\right)\gamma\left(1-\lambda\right)\sigma_z^2-\lambda\sigma_b^2\right]\left[\lambda b_i+\left(1-\lambda\right)z_t+v_{jt}\right]}{\lambda^2\sigma_b^2+\left(1-\lambda\right)^2\sigma_z^2+\sigma_v^2}+\frac{1}{2}Q_{II}\right\}$$

where Q_I and Q_{II} are functions of the model's parameters. These two expectations can be substituted into (21); by solving it with respect to N_{jt} or Y_{jt}, we obtain:

$$Y_{jt}=N_I\exp\left\{\theta\frac{\lambda\sigma_b^2+\left(1-\gamma\right)\left(\frac{1}{\theta}-1\right)\left(1-\lambda\right)\sigma_z^2}{\left[1+\gamma\left(\theta-1\right)+\theta\psi\right]\left[\lambda^2\sigma_b^2+\left(1-\lambda\right)^2\sigma_z^2+\sigma_v^2\right]}\left[\lambda b_i+\left(1-\lambda\right)z_t+v_{jt}\right]\right\} \tag{37}$$

where N_I includes only the model's parameters and the unknown stationary output $\bar{Y}$.

As for the computation of the aggregate equilibrium output Y_t, the same procedure of the previous sections can be followed. Note that the term v_{jt} is uncorrelated and normally distributed; hence, when the jth output level from (37) is substituted into the aggregate output expression (25), the factor including the exponential with v_{jt} can be integrated with respect to dj (or di) by using the law of great numbers, separately from the b_is. The result of the integral is then a term which is a function of the model's parameters. Aggregate output is equal to:

$$Y_t=V_I\exp\left\{\frac{\lambda\sigma_b^2+\left(1-\gamma\right)\left(\frac{1}{\theta}-1\right)\left(1-\lambda\right)\sigma_z^2}{\left[\lambda^2\sigma_b^2+\left(1-\lambda\right)^2\sigma_z^2+\sigma_v^2\right]}\left(\frac{\theta\left(1-\lambda\right)}{1+\gamma\left(\theta-1\right)+\theta\psi}\right)z_t\right\} \tag{38}$$

where V_I is a positive constant (which includes $\bar{Y}$). Now match the coefficients of equation (38) with those of the conjecture (in logs) to obtain:

$$\begin{aligned}\bar{y} &= \bar{v}_I\\ 1 &= \left(\frac{\theta\left(1-\lambda\right)}{1+\gamma\left(\theta-1\right)+\theta\psi}\right)\frac{\lambda\sigma_b^2+\left(1-\gamma\right)\left(\frac{1}{\theta}-1\right)\left(1-\lambda\right)\sigma_z^2}{\lambda^2\sigma_b^2+\left(1-\lambda\right)^2\sigma_z^2+\sigma_v^2}\end{aligned}$$

In this case, the second equation provides:

$$\sigma_z^2=\left[\frac{\theta\left(1-\lambda\right)-\left[1+\gamma\left(\theta-1\right)+\theta\psi\right]\lambda}{\theta\left(1+\psi\right)\left(1-\lambda\right)^2}\right]\lambda\sigma_b^2-\frac{1+\gamma\left(\theta-1\right)+\theta\psi}{\theta\left(\gamma+\psi\right)\left(1-\lambda\right)^2}\sigma_v^2$$

and the condition equivalent to (28) is:

$$\lambda\in\left(0;\frac{1}{2+\psi+\varepsilon}\right);\qquad\frac{\sigma_v^2}{\sigma_b^2}<\lambda\left(\frac{\theta\left(1-\lambda\right)}{1-\gamma+\theta\left(\psi+\gamma\right)}-\lambda\right) \tag{39}$$

Recall that σ_b^2 is related to the unconditional average value of the B_is: $E\left(B_i\right)=\exp(\sigma_b^2/2)$.In this more general case, the parameter σ_b hence plays a more authoritative role in the existence of self-fulfilling equilibria: as shown by the inequality in (39), the possibility of a self-fulfilling equilibrium is greater when $E\left(B_i\right)$ is higher.

3. Assume, first, that agents make the conjecture $y_t=\bar{y}+z_t$ and that at the same time it is: $\lambda>\bar{\lambda}=\frac{\theta}{1+\theta+\theta\psi+\gamma(\theta-1)}$. This inequality can be rewritten in this way:

$$\theta\left(1-\lambda\right)-\left[1+\gamma\left(\theta-1\right)+\theta\psi\right]\lambda<0 \tag{40}$$

From (40) the following inequality can be obtained:

$$\left[\frac{\theta\left(1-\lambda\right)-\left[1+\gamma\left(\theta-1\right)+\theta\psi\right]\lambda}{\theta\left(1+\psi\right)\left(1-\lambda\right)^{2}}\right]\lambda\frac{\sigma_{b}^{2}}{\sigma_{z}^{2}}<0<1$$

Assuming for the moment that it is $\sigma_z > 0$. By considering the inequality with upper bound at 1, the previous inequality can be written in this form:

$$\theta\left(1-\lambda\right)\lambda\sigma_{b}^{2}-\lambda^{2}\sigma_{b}^{2}-\gamma\left(\theta-1\right)\lambda^{2}\sigma_{b}^{2}<\theta\left(1-\lambda\right)^{2}\sigma_{z}^{2}+\theta\psi\left[\lambda^{2}\sigma_{b}^{2}+\left(1-\lambda\right)^{2}\sigma_{z}^{2}\right]$$

and, by adding and subtracting the quantity $\gamma\left(\theta-1\right)\left(1-\lambda\right)^{2}\sigma_{z}^{2}+\left(1-\lambda\right)^{2}\sigma_{z}^{2}$ to the left hand side, the inequality reduces to:

$$\theta\left(1-\lambda\right)\lambda\sigma_{b}^{2}-\left(1-\gamma\right)\left(\theta-1\right)\left(1-\lambda\right)^{2}\sigma_{z}^{2}<1+\gamma\left(\theta-1\right)+\theta\psi\left[\lambda^{2}\sigma_{b}^{2}+\left(1-\lambda\right)^{2}\sigma_{z}^{2}\right]$$

and hence to:

$$\frac{\theta\left(1-\lambda\right)}{1+\gamma\left(\theta-1\right)+\theta\psi}\left[\frac{\lambda\sigma_{b}^{2}+\left(1-\gamma\right)\left(\frac{1-\theta}{\theta}\right)\left(1-\lambda\right)\sigma_{z}^{2}}{\lambda^{2}\sigma_{b}^{2}+\left(1-\lambda\right)^{2}\sigma_{z}^{2}}\right]<1$$

This shows that the coefficient of z_t in (26) is smaller than one and hence that the absolute value of the difference between the actual y_t from (26) and $y_t = \bar{y} + z_t$ is always positive.

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