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MAXIMUM ENTROPY ESTIMATOR FOR THE PREDICTABILITY OF ENERGY COMMODITY MARKET TIME SERIES

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A Maximum Entropy Estimator for the Predictability of Energy Commodity Market Time Series

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Abstract— **This paper proposes a novel method for assessing the predictability of energy market time series, by predicting the entropy of the series. According to conventional entropy-based analysis (where the entropy is always ex-post estimated), high entropy values characterize unpredictable series, while more stable series exhibits lesser entropy values. Here, we predict ex-ante the entropy regarding the future behavior of a series, based on the observation of historical data. Our prediction is performed according to the optimum least squares minimization algorithm. Preliminary results, applied to energy commodities, show the efficacy of the proposed method for application to energy market time series.**

Index Terms—**Entropy analysis, market efficiency, energy commodity, energy time series.**

JEL Classification Numbers: **C530, C630, G170, Q470**

I. INTRODUCTION

The understanding of the dynamic behavior of financial market time series (FMTS), and in particular of energy commodity market time series (EMTS), is of great and crucial interest for market operators. The energy marketplace is growing in complexity and in dynamism. Electricity, natural gas, heating oil, crude oil, and emissions are exchange-traded commodities which are subject to frequent price fluctuations on a short- and long-term basis. Extreme levels of price volatility increase energy price risk, generating, in turn, for various companies and institutions, other types of risk In particular, the observation of historical data as well as the analysis of their volatility (and price fluctuations) can be useful indicators of the dynamic characteristics of the series, in order to effectively perform forecasting procedures. In fact, volatility and implicit risk are strictly related with the amplitude of the series fluctuations: high volatility results in large deviations from the mean, hence stating high unpredictability for that series. According to Pincus and Kalman (2004), FMTS may deviate from constancy exhibiting two different behaviors: (*i*) the series is characterized by high standard deviation, and (*ii*) the series show a lot of irregularities. It is really important to discriminate between these two cases because they lead to different conclusions about the predictability of the series. In fact, the degree of variation from the mean is not usually related with unpredictability, while the amount of irregularities drastically affects the further forecasting process, resulting in unpredictable series. For example, if it would be possible to ensure to an investor that the series of future prices would be characterized by a precise sinusoidal pattern (although characterized by high deviation from the mean), then future prices can be planned according to a precise forecasting strategy. An example of a practical forecasting strategy exploited in the presence of sinusoidal patterns is shown in Giunta and Benedetto (2012).

In particular, we can address, as usual, to standard deviation (SD) as a measure of deviation from the mean, while we will use entropy as the metric for evaluating the irregularities and, hence, the predictability of a series. Entropy is a concept borrowed first from classical mechanics, and then from information theory. In mechanics, entropy is used to quantify disorder and uncertainty of dynamical systems, or in other words, is an expression of the randomness of a system (Jaynes, 1965), while in information theory, entropy is considered as a measure of the information content of the series under investigation (Shannon, 1948). In particular, the mathematician C. Shannon in his pivotal work "*A Mathematical Theory of Communications*" (1948) related the concept of entropy also to that of uncertainty, stating that information measures the degree of uncertainty exercised by the source in selecting the message to transmit (that is, here, the uncertainty of the FMTS behavior in the future). In practice, information is the removal of uncertainty: high values of the Shannon entropy results in an unpredictable series, while lower values means less uncertainty and hence a more predictable behavior of that series.

This paper uses entropy analysis, under the meaning borrowed from information theory, to study the predictability of FMTS. There are plenty of works exploiting the concept of entropy applied to analysis of financial markets time series. For example, the validity of the entropy approach in analyzing financial time series is demonstrated by the work of (Darbellay and Wuertz, 2000). Then, in the work by (Pincus and Kalman, 2004), an empirical method for evaluating the entropy of a series is proposed, namely the *approximate entropy (ApEn)*. In particular, the authors use the approximate entropy technique as a marker of market stability, with rapid increases possibly foreshadowing significant changes in a financial variable. Entropy has been also used to quantify efficiency in foreign exchange markets (Oh *et al*., 2007) and stock markets (Risso, 2008, 2009; Zunino *et al.*, 2009; Gradojevic and Gencay, 2011). Recently, studies focusing on energy commodity markets have been carried out under the entropy-based approach. As an instance, an entropy analysis of crude oil price dynamics is revealed in Martina *et al*. (2011), while evidences from informational entropy analysis in evaluating the efficiency of crude oil markets were discussed in Ortiz-Cruz *et al*. (2012) Then, in (Kristoufek and Vosvrda, 2014) a market efficiency index based on the ApEn metric is discussed for application to several energy commodities. However, all the aforementioned works evaluate the entropy of historical data and, applying *ex-post* considerations, try to declare the predictability of the series, i.e. they implicitly assume that the series under investigation are characterized by a stationary behavior. This means that they suppose that the past statistical features of the analyzed series remain unaltered also in the future.

In this paper, we move further by proposing an algorithm to predict the entropy regarding the future behavior of FMTS, (and in particular of energy commodity market time series), based on the observation of historical data. We do not estimate the entropy of the analyzed series; rather, we predict the entropy of the next time interval of the series. Moreover, we remove the assumption of stationary series, and we only assume that past statistical features influence in some ways future behaviors. Then, according to the conventional entropy analysis, see for example (Pincus and Kalman, 2004; Martina *et al*., 2011), and (Ortiz-Cruz *et al*., 2012), if we predict high entropy values we are probably facing with unpredictable series. Conversely, more stable FMTS exhibits lesser entropy predicted values. Our algorithm exploits the concept of entropy under an information theory viewpoint, recalling the *maximum entropy theory* to evaluate the entropy estimation. In addition, our prediction is performed according to optimum prediction methods, i.e. following the least squares minimization scheme.

The remainder of this work is organized as follows. Section II discusses the basic frameworks about energy market –based entropy analysis. The first half of the section is dedicated to the approximate entropy method, while in the second half the maximum entropy theory is depicted. Then, our proposed entropy estimator is shown in details in Section III, with all the mathematical derivations. Section IV contains the results and discussions about the application of our method to FMTS, in particular to the energy commodity markets Finally, our conclusions are depicted in Section V.

II. Energy Market-Based Entropy Analysis

II.A. Approximate and Multiscale Entropy

Conventional entropy theories are usually related to infinite data series, corresponding to an infinitely accurate precision and resolution for entropy evaluation (Dorfman, 1999). However, practical data are finite time series data, sampled with a sampling rate T_s and characterized by limited resolution. The problem is that accurate estimation of the series entropy requires a big amount of data to be processed, and the results will be greatly influenced by the system noise. To overcome these limitations, (Pincus, 1991) introduced the approximate entropy (ApEn) method, to numerically quantify the entropy content of a finite time series, as a measure of the regularity of the series itself (see Fig. 1). The regularity of the series clearly reflects in the predictability of the series, (Pincus *et al*., 1991). ApEn is able to obtain the entropy estimation by modifying an exact regularity statistic, namely the maximum entropy (or *Kolmogorov-Sinai* entropy). ApEn was initially developed to analyze medical data, such as heart rate in (Pincus *et al*., 1991), and to study physiological time-series in (Pincus and Goldberger, 1994). Later, ApEn spread its applications also in finance, (Pincus and Kalman, 2004).

The ApEn computations are conceptually simple and are based on the likelihood that templates in the time series which are similar remain similar on next incremental comparisons, (Martina *et al*., 2011). In other words, the presence of repetitive patterns of fluctuation in a time series renders it more predictable than a time series in which such patterns are absent. Hence, a time series containing many repetitive patterns has a relatively small ApEn, while time series with large ApEn should have high irregular fluctuation. According to (Pincus and Kalman, 2004), ApEn needs two input parameters to be specified in order to evaluate the approximate entropy of a given time series: a block or run length m, and a tolerance window r. Then, the ApEn procedure first measures the logarithmic frequency that runs of patterns that are close (within the tolerance window r) for m contiguous observations remain close (within the same tolerance r) on the next incremental comparison. A detailed and more formalized definition of the ApEn method is given in (Pincus, 1991). Widely used values for the two input parameters are $m = 1$, $m = 2$, and $r = 20\%$ of the standard deviation of the analyzed time series. ApEn can be designed to work for small data samples (< 50 points) and can be applied in real time, but on the other hand it is heavily dependent on the record length and is uniformly lower than expected for short records. In addition, ApEn lacks relative consistency that is, if ApEn of one data set is higher than that of another, it should, but does not, remain higher for all conditions tested.

The work of (Martina *et al*., 2011) introduces a modified version of the ApEn procedure, namely the *Multiscale* approximate entropy (MApEn), in order to overcome the aforementioned limitations of the original ApEn, In particular, (Martina *et al*., 2011) apply the MApEn algorithm to energy commodity markets, to characterize and monitor the dynamics of crude oil prices. They consider the entropy of a price time series as an index of the market complexity: high entropy values are related to less predictable market evolution (high complexity market). They evaluate the approximate entropy are for different time-scales, performing low-pass filtering of the price difference dynamics. One main drawback of this method is that the low-pass filtering introduces correlation in the analyzed time series (and changes the original data) so that the estimation of the approximate entropy is biased by this filtering operation and depends of the considered time-scale. For instance, a simple uncorrelated series should be always characterized by high entropy values. However, applying the MApEN method for high time scales, results in decreasing the complexity of the random signal, since the low-pass filtering removes the most complex dynamics of the input time series (that now paradoxically exhibits lower entropy values).

II.B. Maximum Entropy

Previous characterizations of the maximum entropy spectral density assume that the process is stationary and Gaussian. Let us now define with $Cov(k)$ the autocovariance function of the input random series $x(n)$ of N data, and defined as:

$$Cov(k) = \frac{1}{N}\sum_{i=1}^{N} x(i) \cdot x^*(i-k) - |\mu|^2 \tag{1}$$

where k = -N, …, +N, $x^*(n)$ stands for complex conjugate, i = 0, ±1, ±2, …, and the mean μ is expressed by:

$$\mu = \frac{1}{N}\sum_{n=1}^{N} x(n) \tag{2}$$

The autocovariance function can be analyzed in the transformed (frequency) domain, obtaining the power spectral density $S(\omega)$ given by (Oppenheim and Schafer, 1975):

$$S(\omega) = \sum_{k=-\infty}^{\infty} Cov(k) \cdot e^{-j\cdot\omega\cdot k} \tag{3}$$

Now, according to (Edward and Fitelson 1978; Chol and Cover, 1984), the entropy rate h is given by:

$$h = \frac{1}{2} ln(2 \cdot \pi \cdot e) + \frac{1}{4\cdot\pi} \cdot \int_{-\pi}^{\pi} ln\,(S(\omega)) \cdot d\omega \tag{4}$$

where $ln(\cdot)$ is the natural logarithm. It is now interesting to underline that the entropy of a finite segment of a stochastic process is upper-bounded by the entropy of a segment of a Gaussian random process, according to (4). This means that a white (i.e. uncorrelated) time series is characterized by the maximum entropy, i.e. it is obviously unpredictable (Chol and Cover, 1984). Lower entropy values result in more predictable time series. Hence, again the entropy can be used as an indicator of the time series predictability.

III. PROPOSED MAXIMUM ENTROPY ESTIMATOR (MEE)

III.A. Motivations

The novelty of our approach is that we now estimate the entropy of the next (future) time interval of the series according to the maximum entropy theory, see eq. (4), by predicting the autocovariance function. The ApEn as well as the MApEn methods estimate the entropy of the observed series, hence making *ex-post* considerations about the predictability of the series itself. Conversely, our approach allows us to make some *ex-ante* considerations (based on the historical observed data) by predicting the autocovariance, and then estimating the entropy, of the series in the next time interval. Moreover, the conventional methods implicitly assume the stationarity of the series, i.e. they assume that future values of the series are characterized by the same behavior observed in the past. Here, we completely remove this statement, assuming that future values can be obtained as a combination of past observed values. Then, in full accordance with the ApEn-based methods, if the predicted entropy is high, this means that the series under investigation would be characterized by high unpredictability. On the contrary, if we estimate a lower entropy value, the series would be characterized by low irregularities, hence resulting in a more predictive behavior for the series.

The starting point of our maximum entropy estimation (MEE) method is the prediction of the autocovariance of the series in the next (unknown) time interval. Once we have the predicted autocovariance, we can easily obtain the power spectral density, see eq. (3), and finally estimate the unknown entropy, see eq. (4). We exploit the basic theory of the optimum linear prediction in order to predict the autocovariance of the series. Linear prediction is a mathematical operation where future values of a discrete-time signal are estimated as a linear function of previous (known) samples. In the case of our interest, future values of the autocovariance are hence estimated as a linear combination of past (observed) autocovariance values. The past autocovariance values are weighted and linearly combined by means of a number p of prediction coefficients.

The outputs of the optimum linear predictor of order p are p prediction coefficients. The Levinson-Durbin recursion is used to solve the equations of the auto-regressive (AR) prediction coefficients that arise from the least-squares formulation (Oppenheim and Schafer, 1975). In practice, the p AR coefficients are chosen as the best coefficients that minimize the prediction error, i.e. the difference between the observed real value and the predicted one (Jackson, 1989). The order of the predictor (i.e. how much of the past story should be taken into account to evaluate the future) drastically affects the performance of the method, in terms of both computational complexity and accuracy of the prediction, as it will be shown in the results' Section.

The inputs of the optimum linear predictor are p autocovariance sequences. This means that, first, we must divide the input series into a number K of consecutive blocks (with $K \geq p$), and for each block we must evaluate the autocovariance, according to eq. (1). Then, only p autocovariance sequences over K (the most recent ones) are used as the inputs of the optimum linear predictor. We choose the most recent blocks as they are probably the blocks that most influence the future behavior of the series. Finally, these p blocks are weighted and linearly combined by the p optimum prediction coefficients, in order to obtain the predicted autocovariance. Finally, the entropy is evaluated. The next subsection depicts in details the algorithm behind the MEE technique.

III.B. Rationale of the MEE method

Given a time series $x(n)$ of length N samples, i.e. $n = 1, 2, \ldots N$, the proposed MEE technique works accordingly to the following steps (see Fig. 2):

1. The N samples of $x(n)$ are divided in K blocks, each of length $M = N / K$ samples.

2. The mean is estimated for each i-th block, according to the following:

$$Mean_i = \frac{1}{M}\sum_{j=1}^{M} x_i(j) \tag{5}$$

where $x_i(j)$ stands for the j-th sample of the i-th block, with $i = 1, \ldots, K$ and $j = 1, \ldots, M$.

3. The mean of the i-th block is subtracted from the next i-th block, so that the blocks are zero-mean series:

$$y_i(j) = x_i(j) - Mean_i \tag{6}$$

This step is required because the object of the maximum entropy analysis needs to be a zero-mean series (Zhang and Xu, 2010).

4. Now, the autocorrelation function, $C_{y,i}(\cdot)$, of each block (i.e. of each sequence $y_i(\cdot)$, with $i = 1, 2, \ldots, K$) is evaluated according to the following:

$$C_{y,i}(k) = \frac{1}{M}\sum_{j=1}^{M} y_i(j) \cdot y_i^*(j-k) \tag{7}$$

As said before, the input of the maximum entropy analysis must be a zero-mean series. Hence, we have now to evaluate the mean of each autocorrelation block, and subtract these means blockwise. In practice, we are evaluating the autocovariance function of each sequence instead of the autocorrelation, according to the following:

$$Cov_{y,i}(k) = \frac{1}{M}\sum_{j=1}^{M} y_i(j) \cdot y_i^*(j-k) - |\mu_i|^2 \tag{8}$$

where $y^*()$ means complex conjugate, and μ_i is the mean of the i-th block of autocorrelation.

5. Now, the K autocovariance functions become the input of the optimum linear predictor of parametric order p. The outputs of the optimum linear prediction step are p prediction coefficients that are now used to estimate the autocovariance of the next block that is the block of which we want to evaluate the entropy. Let us now define with $\widehat{Cov}_{y,K+1}(k)$ the predicted autocovariance sequence of the (K+1)-th block. This sequence is evaluated according to the following:

$$\widehat{Cov}_{y,K+1}(k) = \sum_{b=0}^{p-1} a_b \cdot Cov_{y,K-b}(k) \tag{9}$$

where $Cov_{y,K-b}(k)$ are the linearly combined (K - b) previously observed blocks and a_b are the AR coefficients.

6. Now, according to the theory described in Section II.C, the predicted autocovariance sequence is first transformed in the frequency domain, see eq. (3), obtaining the PSD of the analyzed block. Then, the entropy of the (K+1)-th block is estimated according to the maximum entropy approach, see eq. (4).

IV. RESULTS AND DISCUSSIONS

IV.A. Validation of the MEE method

In order to validate the proposed method as a technique for assessing the predictability of a series, we have matched the results obtained via the MEE method versus the ApEn technique. In particular, we refer only to ApEn as the conventional entropy estimation technique since it is a biased estimator of the true approximate entropy, as well documented by (Pincus, 1995). In addition, we opt to match our results with this method since, as stated by (Kristoufek and Vosvrda, 2014), ApEn is the only entropy measure which is well bounded and thus scalable to a closed interval. In particular, two different controlled experiments were conducted. In the first case (see Fig. 3), we have considered an input *white* Gaussian series (i.e. a completely random series) of unitary SD and zero-mean. Then, we have estimated, by the ApEn method, and predicted, via the MEE approach, the entropy of this uncorrelated series, varying the number of analysed samples. According to the maximum entropy theory, this input series must be characterized by the worst predictability, and hence it should always exhibit the higher entropy. In particular, the ApEn method has been tested varying the number of contiguous observations (m = 1, 2, 5), while the tolerance window length r maintained constant (and equal to 20% of the input SD). In addition, the MEE method has been tested with different prediction orders, (p = 5, 10, and 20) corresponding to different numbers of AR coefficients (i.e. 5, 10, and 20 AR coefficients, respectively). Increasing the prediction order in the MEE approach increases the computational complexity of the method but, at the same time, increases the preciseness of the entropy prediction. However, Fig. 3 shows that the curves referring to the MEE approach are almost overlapping, thus confirming the effectiveness of our method in providing good entropy estimations even with a small number of AR coefficients. In particular, since 10 AR coefficients (i.e. p = 10) are still enough to correctly predict the entropy of the input series, with a reasonable increasing of the system computational complexity (the MEE method must resolve a system of 10 unknowns in 10 independent equations), in the following we focus only on this case for the MEE method. Conversely, increasing the value of the parameter m for the ApEn method decreases the accuracy of the estimation provided by this technique. In fact, it is like as we force the ApEn to find some correlations that are not present in the input series. It is clearly visible from Fig. 3 that the ApEn method with m = 2, 3 provide bad estimates of the entropy, thus confirming the sensitivity of the conventional approach to the values of the input parameters, that must be exactly chosen to let the algorithm perform better. In fact, in the cases of m= 2 and m = 3 the ApEn method needs more samples to provide efficient entropy estimations.

The second experiment was conducted using a *colored* input series, i.e. a series in which some correlations between samples is present. In particular, the input series is constituted by an harmonic series plus a white series. The two series has been summed up varying the powers (equal to the square of the SD) of the analyzed series. Let us define with signal to noise ratio (SNR) the ratio between the power of the harmonic series to the power of the noise (white) series, and expressed in linear scale. Then, the linear SNR is measured in dB, according to the following:

$$SNR_{dB} = 10 \cdot log_{10}(SNR) \tag{6}$$

The SNR has been varied in the interval [-20, +20] dB. This means that when the SNR of the input series is equal to -20 dB, then the series is characterized by a predominant white component: hence the series is hardly predictable. Increasing the SNR means that the harmonic component of the series becomes predominant versus the noise, stating that the series is more and more predictable. In conclusion, in evaluating the entropy versus SNR, we expect to estimate higher entropy values for low SNR, and lower entropy values for higher SNR. This behavior is depicted in Fig. 4, where the MEE method (with p = 10) and the ApEn (with m = 1) are shown. In addition, the input series is also illustrated in some cases of interest, i.e. for an input SNR = -20, 0, and +20 dB. It is interesting to underline that the series, that is highly unpredictable for SNR = -20 dB, exhibits the higher entropy, while the entropy decreases in increasing the SNR, as increasing the predictability of the series itself, as shown in the figure.

IV.B. Application to Energy Market Commodities

Several simulation trials were performed to validate the proposed entropy prediction method for application to energy commodity market time series analysis. We have analysed daily prices of two different energy commodities (Brent Crude Oil and WTI oil prices) in the period between May 20, 1991 and August 14, 2012. The time series were obtained from http://www.quandl.com. Then, we have applied both the ApEn metric and the MEE method in order to verify if the entropy (that is *ex-post* estimated by ApEn and *ex-ante* predicted by MEE) and its variations can be related to the series predictability. In particular, we are going to show that some entropy peaks are strictly connected to some socio-political events, that strongly affect the diversity of the energy commodities under investigation.

Fig. 5 shows the annual entropy variations obtained with both the conventional ApEn technique and the new MEE method for the Brent Crude Oil prices. The ApEn method estimates the entropy ex-post (i.e. we need all the samples of that current year to estimate the entropy of that year), while the MEE approach predict the entropy ex-ante (i.e. we only need the past samples to predict the entropy of the next time interval). In particular, Fig. 5 reports here the entropy estimated via the ApEn technique and obtained with m = 1, and a value of r = 20% of the SD of the series. Moreover, the MEE method is also depicted in Fig. 5, with p = 10. Fig. 5 clearly shows that some peaks of the entropy pattern coincide with the outbreak of some major events. For example, we have some entropy peaks in correspondence to critical events, such as the 1991 Gulf War, 2001-9-11 terrorist attacks and the Lehman Brothers bankruptcy. This suggests that major financial and socio-political events strongly affect the diversity of the Brent crude oil market.

Then, a similar analysis has been conducted also for a different energy commodity, the series of WTI Oil prices. Fig. 6 depicts the annual entropy variations obtained with both the two methods for the WTI Oil prices. In this case, the correspondence between the entropy peaks and some socio-political events is less evident. This is because, as also stated by (Kristoufek and Vosvrda, 2014), the WTI oil series is characterized by a more unpredictable behaviour than the Brent crude oil series, meaning that the WTI oil process series is less predictable than before.

V. Conclusion

A novel technique for predicting the entropy of a time series has been here devised for possible application to energy commodity analysis. In particular, we have applied our prediction technique to energy market time series, matching the conventional approximate entropy estimation method. Preliminary results have confirmed the effectiveness of this computational approach for application to energy-market analysis. Further researches will be devoted to fully characterize the performance of this new prediction method.

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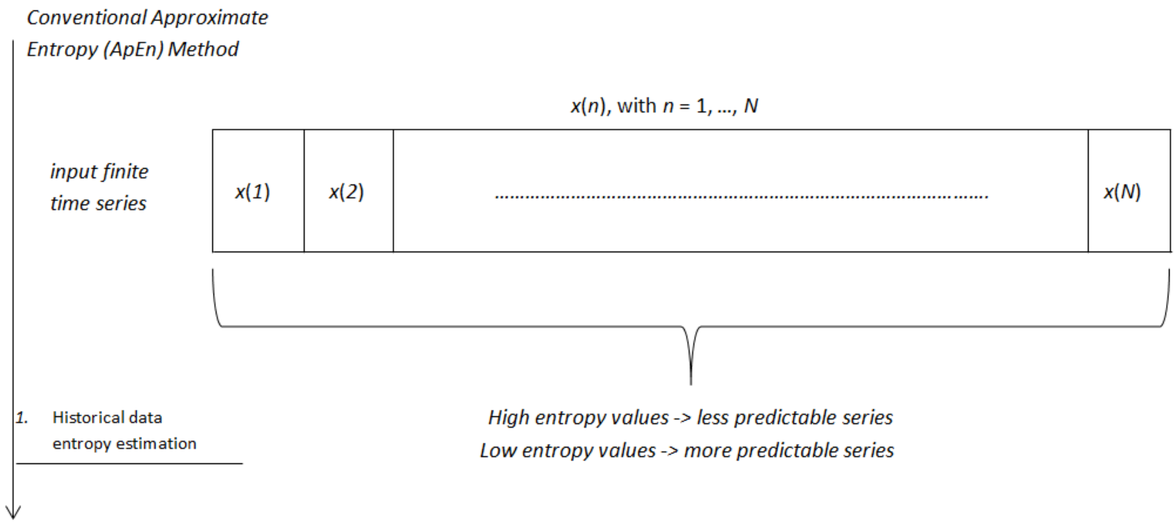

Fig. 1. Flow diagram of the ApEn estimation method.

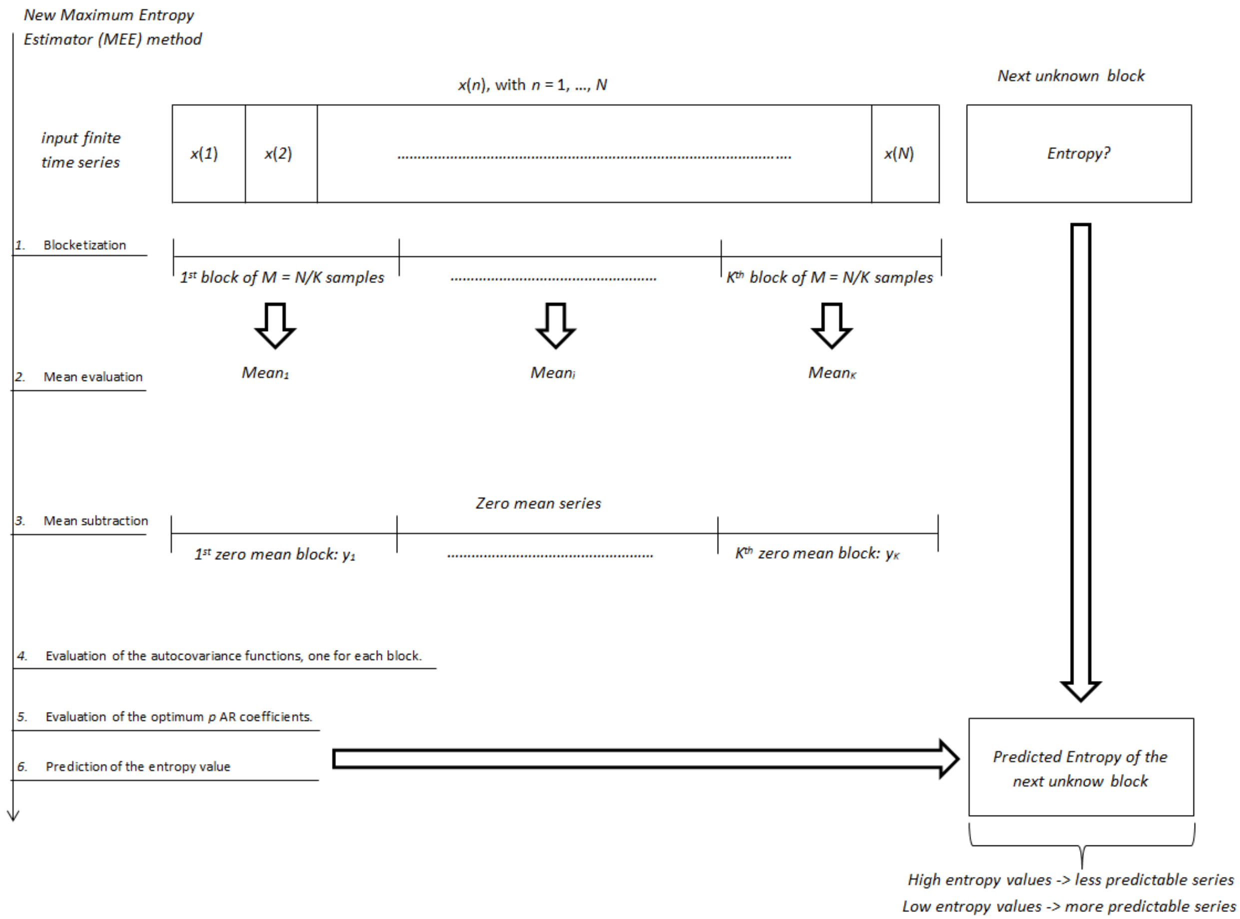

Fig. 2. Flow diagram of the maximum entropy estimation method.

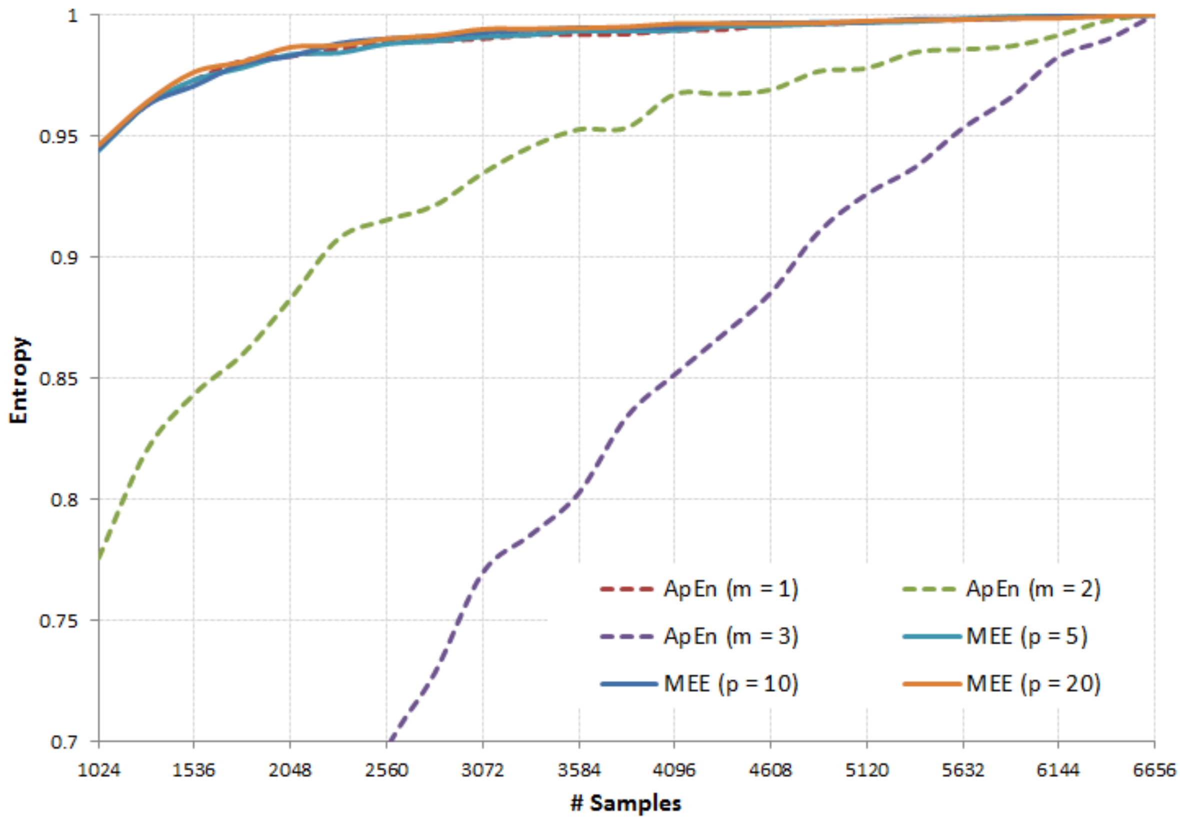

Fig. 3. Normalized (to 1) entropy variations of a white (or uncorrelated) random series vs the number of samples.

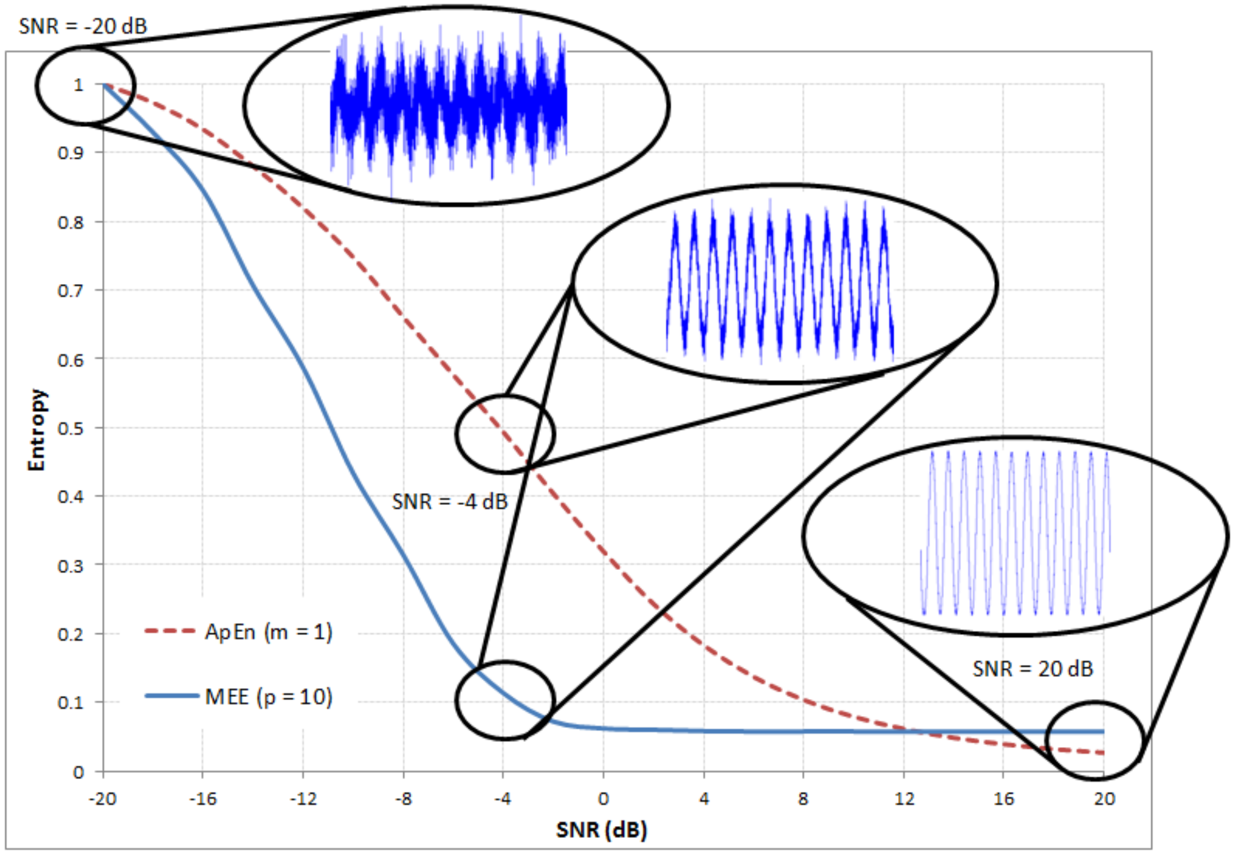

Fig. 4. Normalized (to 1) entropy variations of a colored (or correlated) series vs SNR values.

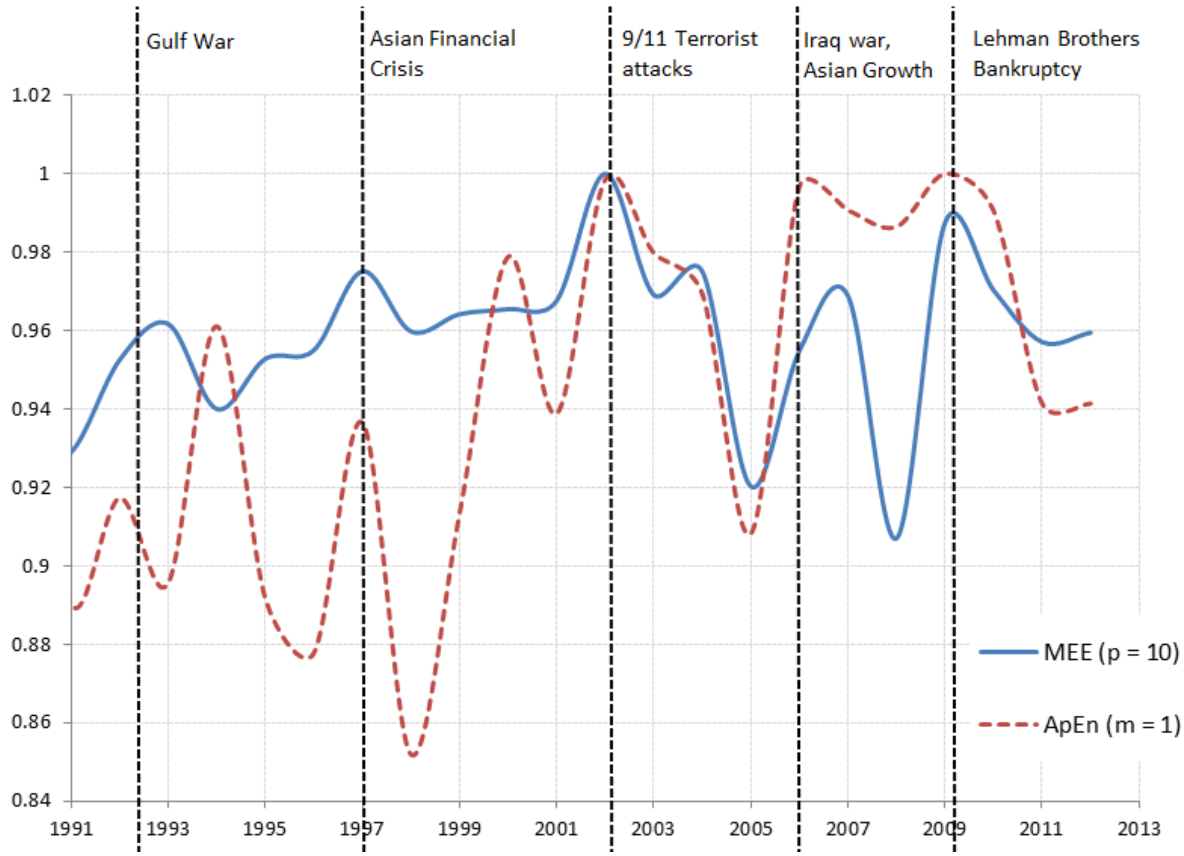

Fig. 5. Annual normalized (to 1) entropy variations of the Brent Crude oil prices.

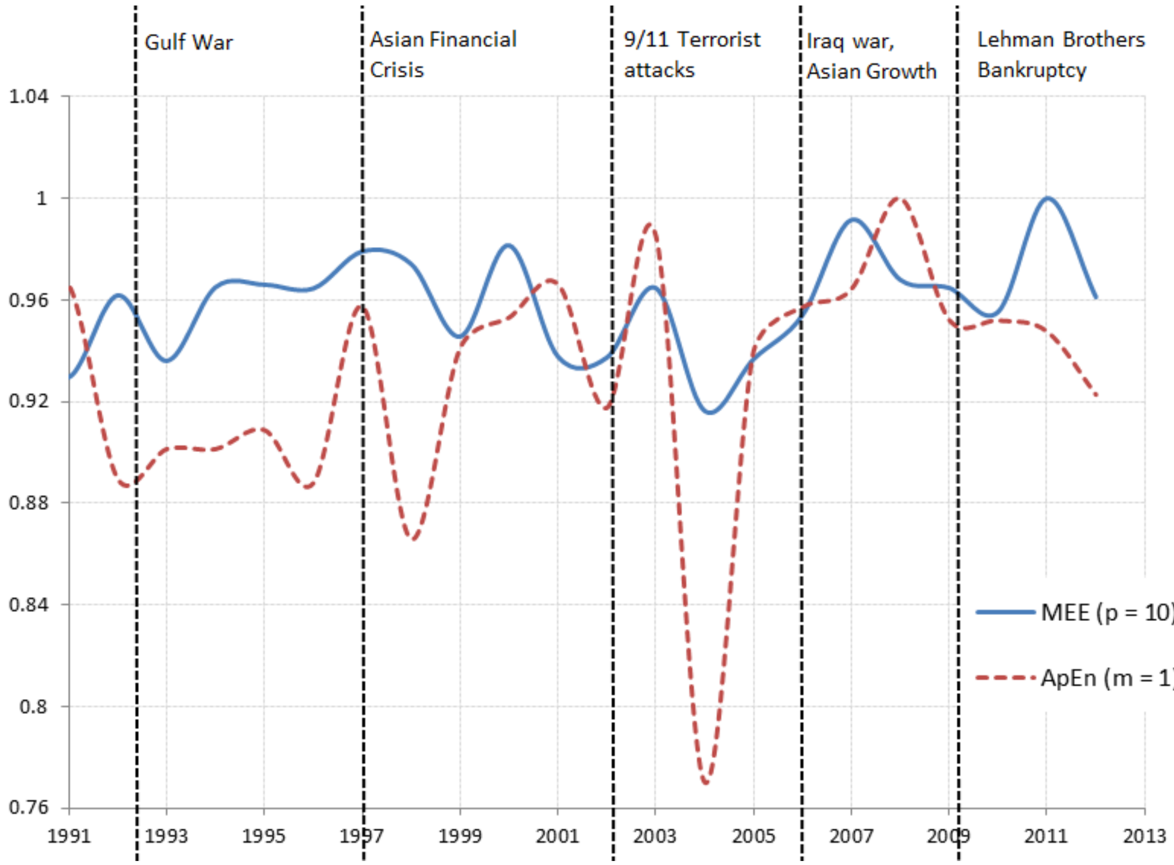

Fig. 6. Annual normalized (to 1) entropy variations of the WTI oil prices.