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DIPARTIMENTO DI ECONOMIA

"BUTTERFLY EFFECT" VS CHAOS IN ENERGY FUTURES MARKETS

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"Butterfly Effect" vs Chaos in Energy Futures Markets.

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Abstract

In this paper we test for the sensitive dependence on initial conditions (the so called "butterfly effect") of energy futures time series (heating oil, natural gas), and thus the determinism of those series. Unlike previous studies, we test for the time series for sensitive dependence on initial conditions, introducing a coefficient that describes the determinism rate of the series and that represents its reliability level (in percentage). The introduction of this reliability level is motivated by the fact that time series generated from stochastic systems also might show sensitive dependence on initial conditions. The reliability level obtained for the NYMEX energy futures considered here is always approximately 50% and this means that the stochastic component and the deterministic one turn up approximately in the same proportions. Such a tangible presence of a stochastic component does not warrant strong evidence of chaotic behaviour.

JEL classification: C450; C530; D400; Q470.

1 Introduction

In recent decades, chaos theory has been considered in various scientific fields such as economics , finance, physics, and many others. An important reason for the interest in chaotic behaviour is that it can potentially explain fluctuations in many time series which appear to be random. In

Keywords: nonlinear dynamics, chaos, butterfly effect, energy futures.

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particular, as for financial markets, evidence on deterministic chaos would have important implications for regulators and short-term trading strategies. The question is whether such random-looking data is really random or it is completely deterministic. If it is completely random, then its behaviour is not predictable anyway; otherwise, it is possible to predict a deterministic system on short periods of time (instead, long prediction is impossible, due to instability of chaotic systems). Hence, this distinction provides the predictability degree of the analyzed system.

Chwee (1998) tests for the presence of chaos using the NYMEX 1-month, 2-month, 3-month, and 6-month daily natural gas settlement prices, from April 1990 to September 1996. In doing so, he uses the BDS statistics and the Lyapunov spectra to determine to what degree futures data resemble a chaotic system. The results fail to provide significant evidence of deterministic chaos. Apostolos Serletis (1999) test for deterministic chaos in seven Mont Belview, Texas hydrocarbon markets, using monthly data from 1985:1 to 1996:12 (the markets are those of ethane, propane, normal butane, iso-butane, naphtha, crude oil, and natural gas). In their paper they estimate the largest Lyapunov exponent, finding an evidence of chaotic process. Panas and Ninni (2000) investigate chaotic structure in daily price data for two major petroleum markets, namely those of Rotterdam and the Mediterranean. The sample consists of the daily prices of different oil products from 4 January 1994 to 7 August 1998, resulting in 1161 observations. All prices were collected from OPEC. The main empirical results obtained by Panas and Ninni's analysis are summarised in Table 5 on Panas and Ninni (2000). The criteria and methods used here are: correlation dimension; entropy; maximal Lyapunov exponent; Eckmann-Ruelle condition; Brocks or residual test theorem; BDS statistic test. They show "strong evidence of chaos in a number of oil products considered". Adrangi et al. (2001) investigate the presence of low-dimensional chaotic structure in crude oil, heating oil, and unleaded gasoline futures prices from the early 1980s. Daily returns data from the nearby contracts are diagnosed employing correlation dimension test, the BDS test and Kolmogrov entropy. While they find "strong evidence of non-linear dependence in the data, the evidence is not consistent with chaos". Saeed Moshiri (2006) examine daily crude oil futures prices from 1983 to 2003, listed in NYMEX; they test for chaos embedding dimension, BDS, Lyapunov exponent, and neural networks tests, finding a negative evidence of chaos. Kyrtsou et al. (2009), analyze five energy products (crude oil, gasoline, heating oil, propane, and natural gas) over the period from 1994 to mid-January 2008. They estimate the dominant Lyapunov exponent, which is negative and in every case they reject the null hypothesis of chaotic behavior. Using both metric (correlation dimension and Lyapunov exponents) and topological methodologies (recurrence plot analysis), Barkoulas et al. (2012) consider a data set which consists of daily oil spot prices covering the period 1.2.1985 - 8.31.2011. Applying the "metric methodologies", they conclude that both the correlation dimension and the Lyapunov exponents show no chaotic tendencies in the oil market and that "the test results from both metric and topological methodologies suggest that oil spot prices are the measured footprint of a stochastic rather than a deterministic system" (pp. 585 Barkoulas et al. (2012)). Matilla-Garca (2007) studies chaotic nature of three energy futures series: natural gas, unleaded gasoline and light crude oil. He investigates the presence of chaos through the largest Lyapunov exponent, finding its "evidence in futures returns". In his conclusions, Matilla-García writes: "A natural question arises: Does evidence of chaos depend on the test procedure used by the researcher? This question is left for future research".

Broadly speaking, one uses the term "chaos" as a synonym of the sensitive dependence on initial conditions, which is the most popular property of a chaotic system, maybe for its intuitive meaning: tiny differences become amplified. The shorthand is the "butterfly effect", a term that has inspired novels and movies. (Let's remember, for example, Ian Malcolm, the mathematician of Jurassic Park.). This property has been introduced for the first time in a formal definition of chaos by Devaney et al. (1989) [1]. According to Devaney, the ingredients of chaos are: sensitive dependence on initial conditions; topological transitivity; density of periodic points. Nevertheless, all the definitions of "chaos" in economic and financial literature, in particular all the papers that we have cited above (Chwee (1998), Apostolos Serletis (1999), Panas and Ninni (2000), Adrangi et al. (2001), Saeed Moshiri (2006), Matilla-Garca (2007), Kyrtsou et al. (2009); see Barkoulas et al. (2012) for the "metric methodologies"), only refer to the first property introduced by Devaney, which represents, in some way, an "experimental" definition of chaos. Even though it can be checked numerically, it is not equivalent to chaos and might be misleading (see also Yousefpoor et al. (2008) and the counterexample 3.3 in Martelli et al. (1998)). Moreover, it can be shown that the butterfly effect is redundant with the other two conditions of Devaney's definition, meaning that it is implied by the other two conditions. All these questions are not negligible since also some time series generated from stochastic systems might show sensitive dependence on initial conditions Tanaka et al. (1996), Ikeguchi and Aihara (1997), Tanaka et al. (1998).

The butterfly effect entails that, if two initial conditions have a small difference δx, their difference after time δt will be $\delta x e^{\lambda \delta t}$ with $\lambda > 0$, that is, with exponential separation. Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for such dynamical systems, making long-term prediction impossible in general. Lyapunov exponents determine the rate of divergence

[1] In literature, there are other definitions of chaos, as for example Li and Yorke (1975), Touhey (1997), Martelli et al. (1998), Wiggins (2013). Some of them are equivalent to Devaney's one, some others are, generally, not easily applicable.

or convergence of initially nearby trajectories in the phase space Kodba et al. (2004), Ott (2002), Schuster and Just (2006), Strogatz (2014). The most important observation is that the largest Lyapunov exponent, denoted as λ_{max}, uniquely determines whether the system shows butterfly effect or not. If $\lambda_{max} > 0$ two initially nearby trajectories of the attractor diverge exponentially fast as time progresses, showing a butterfly effect. Lyapunov exponents represent the rate at which the system creates or distorts information. In fact, let us consider two time series, which have different largest Lyapunov exponents, respectively $\lambda^1_{max} > 0$ and $\lambda^2_{max} > 0$. If $\lambda^1_{max}/\lambda^2_{max} > 1$, then the greater this rate, the faster the propagation of the error in the first time series (compared to the second one).

The aim of this paper is twofold. First, we reread existent works in the literature of energy markets enlightening the role of sensitive dependence on initial conditions in chaos definition: the mathematical definition of chaos recalled here (according to Devaney et al. (1989)) is helpful to prevent us from misleading results about ostensible chaoticity of the returns series. Second, we test for sensitive dependence on initial conditions introducing a coefficient κ that describes the *determinism rate* of the analyzed time series, which represents, in percentage, its reliability level. The introduction of this reliability level is motivated by the fact that, as we have already said, time series generated from stochastic systems might show sensitive dependence on initial conditions.

Also in order to give an answer to Matilla-García's conclusions, we can state, according to the above considerations, that experimental results should be independent from the test used to obtain them: the basic question is whether the treatment of the data can influence the results of the tests. Chaos tests can be conducted on both raw as well as filtered data. Empirical analysis of identification of chaotic structure of time series necessarily raises the question of filtering (the transformation of raw data prior to its analysis), because the filtered data may give a false indication of chaos. Panas and Ninni (2000) compare filtered and raw data, pointing out that the obtained results on the butterfly effect are approximatively the same, even if, generally, filtering may affect the dimensionality of the original data (see Chen (1993), as reported also in Panas and Ninni (2000)) and the filtered data may mimic a chaotic behaviour. However, in our approach we have used only raw data and we have left the question of filtering for future works.

The paper is organized as follows. In Section 2 we consider the implications of chaos in energy futures markets, and introduce one of the most popular definitions of chaos, the Devaney's definition, suitable to our purposes. In Section 3 we explain the results of the paper. Section 4 is devoted to the conclusions while the Appendix 1 (Section 5) is devoted to the mathematical methodologies of the tests we conducted here, in comparison with some others that appear in the literature. Appendix 2 (Section 6) contains the tables concerning the numerical evaluations.

2 Implications of “chaos” in energy futures time series

In literature (Devaney et al. (1989), Kodba et al. (2004), Ruelle (1989), Wiggins (2013)), chaos is considered as an alternative to randomness for systems of “strange” behaviour. One of the most famous (and simplest) examples of chaotic system is the *logistic map* (that underlies the well-known *logistic growth model*); it is expressed by the relation

$$x_{n+1} = r\, x_n(1 - x_n)$$

where x_n can represents the ratio of existing population to the maximum possible population and varies between zero and one. The parameter $r > 0$ represents the exponential growth rate of population. The logistic map is *chaotic* (later, we will focus on the meaning of “chaos”) for $r > 2 + \sqrt{5} \approx 4.236$ (Devaney et al. (1989), pp. 31 - 50). The quality of unpredictability[2] and apparent randomness led the logistic map equation to be used as a pseudo-random number generator in calculators Gleick (2011). On the other hand, we can say that a chaotic map is a deterministic map which is able to produce “random looking” data.

Another notable system that has chaotic solutions for certain parameter values and initial conditions and that represents a starting point in the whole literature on chaos, is the *Lorenz system*, Lorenz (1963), a simplified mathematical model for atmospheric convection.

From a technical standpoint, in both cases we start from a mathematical model (one or more equations). Nevertheless, in many actual situations (such as dealing with financial markets and, for our aims, energy markets) a mathematical model might not be available, in which cases work on time series would be needed. Generally, the accurate structural modelling of commodities, and in particular energetic commodities, could be considered to be impossible. Therefore, evidence of chaos could offer some strategies for modelling price behaviour by simply employing the time series of prices.

There are many possible definitions of chaos, ranging from measure theoretic notions of randomness in ergodic theory to the topological approach. In our paper we will focus on the definition of chaos by Devaney et al. (1989) .

Devaney wrote: “To summarize, a chaotic map possesses three ingredients: unpredictability, indecomposability, and an element of regularity. A chaotic system is unpredictable because of the sensitive dependence on

[2] Meaning that chaotic systems are unpredictable in a way that other deterministic systems are not Werndl (2009).

initial conditions. It cannot be broken down or decomposed into two subsystems (two invariant open subsets) which do not interact under f because of topological transitivity. And, in the midst of this random behaviour, we nevertheless have an element of regularity, namely the periodic points which are dense" Devaney et al. (1989). In other words, topological transitivity means that the dynamical system f is, in a sense, indecomposable in simpler systems, and any given region of its phase space will eventually overlap with any other given region. This also means that, taken two points P and Q, their trajectories, initially close together, can suddenly move in completely different directions. An orbit (or a point or a trajectory) that repeats is called a periodic orbit. The density of periodic points implies that every point in the space is approached arbitrarily closely by periodic orbits. This excludes that a chaotic system can be in some way periodic and it also means that, taken two points P and Q, their trajectories, initially far apart, can wind up in almost the same place.

In literature, it's widely used an "experimental" definition of chaos, that takes into account only the first condition of Devaney's definition, the sensitive dependence on initial conditions. There are a lot of numerical tests for this property, but this definition of chaos, as we already said, is not satisfactory. See, for instance, the counterexample 3.3 in Martelli et al. (1998).

Over the years, chaos theory gradually has provided a framework to study some interesting properties of time series. Some widespread tests are: correlation dimension, the BDS test, Kolmogrov entropy, Lyapunov exponent, close returns test, etc (see Section 3 for a survey of these tests and a comparison with our approach). Some of them are not properly chaos tests but can be able to investigate properties like nonlinearity [3]. As pointed out in Yousefpoor et al. (2008), some tests, which are not chaos tests in fact, have been implemented in chaos literature (especially in applied science such as, for our aims, energy markets). A chaotic system must be characterized by some basic properties, while the tests mentioned above focus only on one of these aspects. Although this is not sufficient to ensure the presence of chaos, these tests could be useful to study important properties of system dynamics (such as butterfly effect).

Chaotic time paths have several properties that should be of special interest to commodity market observers, such as the apparent stochasticity of time series that could be generated by deterministic systems, or the butterfly effect.

In particular, since the energy futures time series show an irregular random behaviour that often resembles chaos, we employ the determinism test introduced by Kaplan and Glass (1992), in order to verify whether the stud-

[3] Chaotic dynamics are necessarily non-linear, but there are many examples of non-linear dependencies that are not consistent with chaos, as obtained in Adrangi et al. (2001).

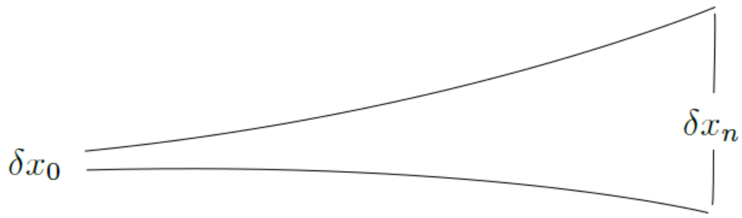

Figure 1: Effect of a small change of initial condition.

ied time series indeed originates from a deterministic system. For this purpose, we use a package written in $C++$ code and developed in Kodba et al. (2004). The importance of this procedure in the framework implemented here will be discussed in Section 3. The idea is to take the *determinism coefficient* κ, obtained by Kaplan and Glass' test, as a measure of the reliability level (in percentage) of test on sensitive dependence on initial conditions. Actually, it is known that time series generated from stochastic systems also might show positive maximal Lyapunov exponents Tanaka et al. (1996), Ikeguchi and Aihara (1997), Tanaka et al. (1998).

The butterfly effect has important implications in forecasting of chaotic time series. Let us consider the time series y_n and assume that there exists a system (g, f, x_0) such that $y_n = g(x_n)$, $x_{n+1} = f(x_n)$, where x_0 is the initial condition at initial time $n = 0$, and where g maps the m-dimensional phase space $\mathbb{R}^m$ to $\mathbb{R}$, and f maps $\mathbb{R}^m$ to $\mathbb{R}^m$. The function f maps an unknown (to the econometrician) dynamics that governs the evolution of the unknown (to the econometrician) state x_0. The econometrician observes y_n. The task is to uncover information about (g, f, x_0) from observations y_n Brock and Sayers (1988). The time series y_n, which we will assume as the data time series under analysis, has a chaotic explanation if x_n is chaotic. The question is whether it is possible to forecast a chaotic series. Intuitively, the butterfly effect usually doesn't allow long-term forecasting of chaotic series. If we change slightly the value of initial point: $x_0 \mapsto x'_0 = x_0 + \delta x_0$, the point x_n at discrete time n will also be changed (see Fig. 1). What may happen is that, when time becomes large, the small initial distance δx_0 grows anyway, and it may grow exponentially fast: $\delta x_n \sim \delta x_0 e^{\lambda n}$, for some $\lambda > 0$. The term δx_n represents the uncertainty induced by perturbations. Hence, fixed δx_0 and λ, it is bounded if $e^{\lambda n}$ is bounded and so if n is as small as possible ($n = 1$ or $n = 2$, for instance): this is the short-term forecasting of chaotic series, which is possible because δx_n is amplified at a finite rate $e^{\lambda n}$. Accordingly, for chaotic time series, if one knows (g, f) and could measure x_n without error, one could forecast x_{n+i} and, thus, y_{n+i} perfectly in the short time (say a few days when dealing with daily data).

3 The empirical results

In this Section we test for sensitive dependence on initial conditions and determinism of the following energy futures series: heating oil (06.03.1979 - 15.05.2014), natural gas (03.04.1990 - 15.05.2014). The time series were taken by NYMEX and were obtained from `http://www.quandl.com`. In particular, we investigate the presence of low-dimensional chaotic structure. Low-dimensional chaos is usually used to refer to dynamics with only one positive Lyapunov exponent Harrison and Lai (1999), while, on the contrary, high-dimensional chaos corresponds to more than one positive Lyapunov exponent. High and low-dimensional chaos in this sense are not linked to the embedding dimension. Although the treatment of this difference is beyond the scope of this paper and will be treated elsewhere, considering a three-dimensional system with the Lyapunov exponent spectrum $(\lambda_1, \lambda_2, \lambda_3)$, it has been shown that $(+, 0, -)$ indicates a chaotic attractor. Hence, if, by definition, a high-dimensional chaotic system has at least two positive Lyapunov exponents, the Lyapunov exponent spectrum is $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (+, +, 0, -)$, and thus the minimal dimension for a high-dimensional chaotic system is $m = 4$. This also means that, for $m < 4$, we can have only the presence of low-dimensional chaos.

In order to investigate the presence of low-dimensional chaotic structure, fixing the embedding dimension $m = 2$ or $m = 3$, for each commodities we analyze the determinism rate κ on varying of τ. We use a package written in $C++$ code, which can be downloaded from M. Perc's Web page [4], as described in Kodba et al. (2004).

To calculate the maximal Lyapunov exponent λ_{max}, we used a Matlab Mat (2015) code due to Wolf et al. and available on Matlab Central File Exchange. It is based on an algorithm presented in Wolf et al. (1985), which estimates the dominant Lyapunov exponent of a 1-D time series by monitoring orbital divergence. The algorithm was distributed for many years by the authors in Fortran and C and subsequently has just been converted to Matlab. This algorithm has known a widespread success in literature: suffice it to say that while we write this paper, the work due to Wolf et al. was cited 6,963 times (s: Google Scholar).

The main empirical results obtained in our analysis are summarised in Tables 1, 2, 3 and 4. In them one uses results on determinism coefficient shown in Appendix. The Tables exhibit Maximum Lyapunov exponents for the cases with higher value determinism coefficient.

[4] M. Perc Web Page, `http://www.matjazperc.com/ejp/time.html`.

m	τ	κ	MLE
2	2	0.501905	2.7159
2	39	0.444272	1.7691
2	4	0.441020	2.5150
2	28	0.433819	2.0525
2	5	0.431399	2.4318

Table 1: Maximum Lyapunov exponent (MLE) for Heating OIL HO1, temporal range 06.03.1979 - 15.05.2014, number of samples 8,825. Case $m = 2$.

m	τ	κ	MLE
3	2	0.537274	2.2768
3	4	0.485819	2.3791
3	3	0.478729	2.6322
3	5	0.469815	2.2892
3	7	0.454737	2.0076

Table 2: Maximum Lyapunov exponent (MLE) for Heating OIL HO1, temporal range 06.03.1979 - 15.05.2014, number of samples 8,825. Case $m = 3$.

m	τ	κ	MLE
2	16	0.493609	12.5925
2	14	0.477181	12.3958
2	2	0.476278	13.9755
2	15	0.459651	11.6615
2	8	0.452906	13.1308

Table 3: Maximum Lyapunov exponent (MLE) for Natural GAS, temporal range 03.04.1990 - 15.05.2014, number of samples 6,042. Case $m = 2$.

m	τ	κ	MLE
3	2	0.565059	12.1163
3	3	0.483286	11.8509
3	16	0.476005	8.9366
3	8	0.473239	10.4472
3	4	0.472915	10.9943

Table 4: Maximum Lyapunov exponent (MLE) for Natural GAS, temporal range 03.04.1990 - 15.05.2014, number of samples 6,042. Case $m = 3$.

These results suggest that there is no strong evidence of sensitive dependence on initial conditions (butterfly effect) for the series considered here because κ is not near to 1.

To better understand the results obtained in our paper, let's take, for example, Table 3. When $m = 2$ and $\tau = 16$ or $\tau = 8$, natural gas time series exhibit positive maximal Lyapunov exponents with determinism coefficient, $\kappa = 0.493609$ and $\kappa = 0.452906$, respectively. Since there are stochastic systems that show sensitive dependence on initial conditions (Tanaka et al. (1996), Ikeguchi and Aihara (1997), Tanaka et al. (1998)), we propose to explain these results on κ as the reliability level (in percentage) of the maximal Lyapunov exponents results. Thus, when $m = 2$ and $\tau = 16$ natural gas time series exhibit positive maximal Lyapunov exponents with reliability level at $\simeq 49\%$: too low to be able to conclude that there is strong evidence of sensitive dependence on initial conditions.

We recall that all the papers concerning energy time series that we have cited in this paper (Chwee (1998), Apostolos Serletis (1999), Panas and Ninni (2000), Adrangi et al. (2001), Saeed Moshiri (2006), Matilla-Garca (2007), Kyrtsou et al. (2009), Barkoulas et al. (2012)) are based on the "experimental" definition of chaos. Among them, Apostolos Serletis (1999), Panas and Ninni (2000) and Matilla-Garca (2007) discover some evidence of butterfly effect. Panas and Ninni (2000) employ the BDS statistical test and the Correlation Dimension test to distinguish determinism by stochastic process; with the same aim, Matilla-García employs the BDS statistical test and the Kaplan test (see Matilla-Garca (2007) for details) while Apostolos Serletis (1999) apply a nonlinear analysis in order to remove any stochastic dependence.

With regard to the futures time series analyzed in our paper, we notice the following differences with respect to other papers.

- Kyrtsou et al. (2009) have observed natural gas and heating oil, over the period from 1994 to mid-January 2008, showing that Lyapunov exponent estimates are negative. They also show the existence of a structure that is partially deterministic. The largest Lyapunov exponents detected in our paper for natural gas and heating oil are positive but the futures time series show a not negligible contribute of stochasticity ($\kappa \lesssim 50\%$ for all the measures).

- As for heating oil, Adrangi et al. (2001), for observations on the range 1/02/85 - 03/31/95, employ correlation dimension test, the BDS test and Kolmogrov entropy, without finding evidence of butterfly effect. In Table 5, we have calculated the *correlation dimension* [5] of NYMEX-heating oil daily series considered here. We notice the absence of saturation, that doesn't provide evidence of chaotic structure, and therefore confirms the results by Adrangi et al. obtained with correlation dimension method (Adrangi et al. (2001), Table 3, p. 416).

- As for natural gas, Matilla-Garca (2007) uses observations from 04/03/1990 to 10/19/2005. He discovers the positivity of the largest Lyapunov exponent while Chwee (1998), examining observations from April 1990

[5] File c2.m from D. Chelidze's home page, `http://egr.uri.edu/nld/software/`.

to September 1996, shows no evidence of butterfly effect from the estimation of the Lyapunov spectra. The range considered from Matilla-García is approximately the same we have considered here (with obviously few years less). Matilla-García, testing for the butterfly effect, employed a method, by Rosenstein et al. (1993), belonging to the same family as the one used here by Wolf Wolf et al. (1985), and therefore his results on the positivity of MLE agree with ours. Since it is possible that Rosenstein's and Wolf's algorithms find positive values for the Lyapunov exponent also for any pure random process, his results are accompanied by a test Fernández-Rodríguez et al. (2005) which sets a deterministic process as null hypothesis, while the alternative hypothesis is that of a stochastic process. However, we point out that there are several systems that have both stochastic and deterministic components, Kaplan and Glass (1992), and a "null hypothesis" could be insufficient to describe these cases. For this reason we have introduced the parameter κ.

- Again, as for natural gas, Apostolos Serletis (1999) examining monthly data from 1985:2 to 1996:12, "test for positivity of the dominant Lyapunov exponent. Before conducting such a nonlinear analysis, the data were rendered stationary and appropriately ltered, in order to remove any linear as well as nonlinear stochastic dependence". They have found evidence of butterfly effect in all natural gas liquids markets. Compared to their paper, in our work doesn't apply any filtering to data.

Moreover, although these studies investigate the presence of both stochastic and deterministic component in time series, they don't provide any estimate of determinism rate existing in the analyzed data.

τ	$m = 5$	$m = 10$	$m = 15$	$m = 20$
5	-0.0841	-0.1377	-0.1840	-0.2123
10	-0.0845	-0.1406	-0.1914	-0.2259
15	-0.0848	-0.1427	-0.2004	-0.2355
20	-0.0852	-0.1452	-0.2076	-0.2428
25	-0.0855	-0.1487	-0.2123	-0.2469
30	-0.0860	-0.1523	-0.2163	-0.2507
35	-0.0869	-0.1569	-0.2203	-0.2649
40	-0.0878	-0.1614	-0.2242	-0.2691

Table 5: The table reports $\log_{10} C(\epsilon)$, where $C(\epsilon)$ is the correlation integral, for the daily prices of NYMEX heating oil (06.03.1979 - 15.05.2014).

4 Conclusions

In this paper we have tested sensitive dependence on initial conditions and determinism of the NYMEX energy futures: heating oil (06.03.1979 - 15.05.2014), natural gas (03.04.1990 - 15.05.2014). Fixing the embedding dimension $m = 2$ or $m = 3$, for each commodity we have analyzed the determinism rate κ on varying of τ, employing a package developed in Kodba et al. (2004) and based on the determinism test introduced in Kaplan and Glass (1992). Afterward, to check sensitive dependence on initial conditions we have selected the largest Lyapunov exponents test, using a Matlab code based on an algorithm presented in Wolf et al. (1985). The main empirical results obtained in the above steps are summarised in Tables 1, 2, 3 and 4.

The introduction of the reliability level κ is motivated by the fact that time series generated from stochastic systems also might show sensitive dependence on initial conditions, and then the employment of only MLE method is not sufficient. The reliability level obtained for the NYMEX energy futures considered here is always approximately 50% and this means that the stochastic component and the deterministic one turn up to be approximately in the same proportions. Such a tangible presence of a stochastic component doesn't allow one to conclude that there is strong evidence of chaotic behaviour. Moreover:

- Being able to determine whether there is chaos is extremely important in order to make predictions in the financial markets and in particular with regard to the energy market.
- The literature does not yet seem to be clear about how you need to test and especially about the tests to be performed, in order to prove (or disprove) the presence of chaotic behaviour.
- We try, with our paper, to go in the direction in which you get to determine whether or not there is butterfly effect.

Different approaches from those employed in this paper, such as the 0-1 test for chaos Gottwald and Melbourne (2004)-Gottwald and Melbourne (2005) or the recurrence plot analysis also used by Barkoulas et al. (2012), deserve further discussions and will be dealt with in next work. It would also be interesting to compare the results obtained in previous works employing the Shannon entropy Benedetto et al. (2015, 2016) (see also Gençay and Gradojevic (2010); Gradojevic and Gencay (2008) for the investigation of the evolution of the aggregate market expectations) with the results obtainable through Lyapunov exponents and Kolmogorov entropy.

5 Appendix 1: Mathematical methodologies of the tests

The sensitive dependence on initial conditions can be checked by Kolmogrov entropy test and the Largest Lyapunov exponents.

The definition of Kolmogorov entropy (or Kolmogorov-Sinai entropy Kolmogorov (1958), Sinaǐ (1959), or metric entropy) K can be found, for example, in Farmer (1982) (eq. 7, p. 370). For systems that show butterfly effect, K is positive[6]. Grassberger and Procaccia Grassberger and Procaccia (1983b) have proposed a quantity K_2 which has the following properties: (i) $K_2 \geq 0$; (ii) $K_2 \leq K$; (iii) K_2 is infinite for random systems; and (iv) $K_2 \neq 0$ for systems which show butterfly effect. Nevertheless, the largest Lyapunov exponents test has been selected here, because of its clearer result. If the largest Lyapunov exponent is positive, then it would imply butterfly effect but if it is negative it would not, while $K_2 > 0$ is only a sufficient condition for butterfly effect.

The existing determinism structure in the analyzed time series can be checked also by the correlation dimension test. The correlation dimension test measures a quantity called *correlation dimension* and has usually been introduced as a test for distinguishing randomness and chaoticity. It distinguishes chaotic series from random series by investigating the correlation dimension behaviour of the data, Gabisch and Lorenz (2013).

Grassberger and Procaccia Grassberger and Procaccia (1983a) Grassberger and Procaccia (1983c) show that correlation dimension D_2 can be evaluated using the correlation integral $C(\epsilon)$, which is defined as the probability that a pair of points chosen randomly with respect to the natural measure is separated by a distance less than ϵ from the attractor. For a stochastic signal, $C(\epsilon)$ scales like ϵ^m for all m. In contrast, $C(\epsilon)$ scales like ϵ^{D_2} for m larger than the attractor dimension, if the signal is generated by a deterministic system (Grassberger and Procaccia (1983c), p. 206).

In other words, one can have the following cases: (i) if the value of $C(\epsilon)$ stabilizes at some value as m increases, then the signal is generated by a deterministic system; (ii) if $C(\epsilon)$ continues to vary as m is raised, then the system is to be regarded as stochastic. When case (i) occurs, usually one tests for butterfly effect of the analyzed time series. Nevertheless, in this way we don't get a measure of stochasticity (or determinism) rate of the time series. How reliable are the results obtained on butterfly effect?

[6]Farmer Farmer (1982), Grassberger and Procaccia Grassberger and Procaccia (1983b) mention "chaos" but they refer to the experimental definition of chaos. In fact, Grassberger and Procaccia, defining the expression of K, cite the paper of Farmer, while Farmer defines "chaotic attractor as any attractor with positive metric entropy" (p. 372, Farmer (1982)) and, as we can read in the same page, K can be expressed as the sum of positive Lyapunov exponents, which constitute a measure of butterfly effect and not of chaos, as previously recalled.

In our paper Kaplan and Glass' algorithm (Kaplan and Glass (1992), Kodba et al. (2004) and references therein) has been preferred to Grassberger and Procaccia algorithm because of its more explicit result. As we have recalled, the determinism coefficient κ is equal to 1 for a deterministic system, while $\kappa = 0$ for a random walk. In a way, κ measures the distance of time series from a deterministic system as well as from a stochastic process. In other words, it measures both the rate of "stochasticity" and that of "determinism" in a time series, while the Grassberger and Procaccia algorithm does not provide such explicit results. Hence, our idea is to take the κ as a measure of the reliability level (in percentage) of the results obtained with the test on butterfly effect.

6 Appendix 2

Fixed $m = 2$ or $m = 3$, in the following Tables we show the numerical values of the determinism coefficient κ on varying of τ.

τ	κ	τ	κ	τ	κ
2	0.501905	16	0.361204	30	0.408889
3	0.419842	17	0.375060	31	0.411305
4	0.441020	18	0.384556	32	0.412225
5	0.431399	19	0.398143	33	0.379679
6	0.392880	20	0.412437	34	0.394530
7	0.395867	21	0.406275	35	0.394847
8	0.406614	22	0.381689	36	0.417282
9	0.415095	23	0.395406	37	0.430625
10	0.391700	24	0.396505	38	0.399006
11	0.415090	25	0.409085	39	0.444272
12	0.395824	26	0.423537	40	0.417778
13	0.375370	27	0.419672	41	0.384796
14	0.407810	28	0.433819	42	0.426004
15	0.408089	29	0.428505	43	0.395696

Table A1: Heating oil, temporal range 06.03.1979 - 15.05.2014, number of samples 8,825, $m = 2$.

τ	κ	τ	κ	τ	κ
2	0.537274	16	0.394520	30	0.367590
3	0.478729	17	0.403045	31	0.404781
4	0.485819	18	0.401586	32	0.415901
5	0.469815	19	0.399474	33	0.404190
6	0.438405	20	0.414770	34	0.412973
7	0.454737	21	0.420266	35	0.414922
8	0.420400	22	0.404316	36	0.438795
9	0.443673	23	0.406564	37	0.401609
10	0.406686	24	0.384466	38	0.414218
11	0.410421	25	0.368297	39	0.395303
12	0.384327	26	0.393778	40	0.422266
13	0.406481	27	0.353556	41	0.399929
14	0.431002	28	0.398153	42	0.381433
15	0.409526	29	0.390039	43	0.406102

Table A2: Heating oil, temporal range 06.03.1979 - 15.05.2014, number of samples 8,825, $m = 3$.

τ	κ	τ	κ	τ	κ
2	0.476278	16	0.493609	30	0.431342
3	0.423472	17	0.432316	31	0.419427
4	0.427403	18	0.407716	32	0.432981
5	0.393378	19	0.417947	33	0.427012
6	0.420013	20	0.417331	34	0.410806
7	0.451157	21	0.437138	35	0.429270
8	0.452906	22	0.411759	36	0.421174
9	0.436177	23	0.402592	37	0.446244
10	0.451256	24	0.411774	38	0.369850
11	0.431307	25	0.418427	39	0.402403
12	0.419933	26	0.378412	40	0.387175
13	0.419721	27	0.412978	41	0.440380
14	0.477181	28	0.401562	42	0.445409
15	0.459651	29	0.433165	43	0.426742

Table A3: Natural gas, temporal range 03.04.1990 - 15.05.2014, number of samples 6,042, $m = 2$.

τ	κ	τ	κ	τ	κ
2	0.565059	16	0.476005	30	0.448111
3	0.483286	17	0.457383	31	0.416498
4	0.472915	18	0.418325	32	0.426808
5	0.448891	19	0.430515	33	0.417782
6	0.420842	20	0.433987	34	0.411756
7	0.459661	21	0.459805	35	0.399251
8	0.473239	22	0.422080	36	0.396002
9	0.431479	23	0.423422	37	0.441582
10	0.439751	24	0.411347	38	0.387324
11	0.421026	25	0.450880	39	0.421272
12	0.403814	26	0.418336	40	0.393928
13	0.427620	27	0.420452	41	0.440154
14	0.446716	28	0.421035	42	0.460074
15	0.455369	29	0.422340	43	0.425142

Table A4: Natural gas, temporal range 03.04.1990 - 15.05.2014, number of samples 6,042, $m = 3$.

References

(2015), *MATLAB version 8.5.0.197613 (R2015a)*. The Mathworks, Inc., Natick, Massachusetts.

Adrangi, Bahram, Arjun Chatrath, Kanwalroop Kathy Dhanda, and Kambiz Raffiee (2001), "Chaos in oil prices? Evidence from futures markets." *Energy Economics*, 23(4), 405–425.

Apostolos Serletis, Periklis Gogas (1999), "The north american natural gas liquids markets are chaotic." *The Energy Journal*, 20(1), 83–103.

Barkoulas, John T, Atreya Chakraborty, and Arav Ouandlous (2012), "A metric and topological analysis of determinism in the crude oil spot market." *Energy Economics*, 34(2), 584–591.

Benedetto, F, G Giunta, and L Mastroeni (2015), "A maximum entropy method to assess the predictability of financial and commodity prices." *Digital Signal Processing*, 46, 19–31.

Benedetto, F, G Giunta, and L Mastroeni (2016), "On the predictability of energy commodity markets by an entropy-based computational method." *Energy Economics*, 54, 302–312.

Brock, William A and Chera L Sayers (1988), "Is the business cycle characterized by deterministic chaos?" *Journal of Monetary Economics*, 22(1), 71–90.

Chen, Ping (1993), "Searching for economic chaos: a challenge to econometric practice and nonlinear tests." In *Nonlinear Dynamics and Evolutionary Economics* (R. H. Day and P. Chen, eds.), volume 217, Oxford University Press New York.

Chwee, Victor (1998), "Chaos in natural gas futures?" *The Energy Journal*, 19(2), 149–164.

Devaney, Robert L et al. (1989), *An introduction to chaotic dynamical systems*, volume 13046. Addison-Wesley Reading.

Farmer, J Doyne (1982), "Chaotic attractors of an infinite-dimensional dynamical system." *Physica D: Nonlinear Phenomena*, 4(3), 366–393.

Fernández-Rodríguez, Fernando, Simón Sosvilla-Rivero, and Julián Andrada-Félix (2005), "Testing chaotic dynamics via Lyapunov exponents." *Journal of Applied Econometrics*, 20(7), 911–930.

Gabisch, Günter and Hans-Walter Lorenz (2013), *Business cycle theory: a survey of methods and concepts*. Springer Science & Business Media.

Gençay, Ramazan and Nikola Gradojevic (2010), "Crash of'87 — Was it expected?: Aggregate market fears and long-range dependence." *Journal of Empirical Finance*, 17(2), 270–282.

Gleick, James (2011), *Chaos: Making a New Science (Enhanced Edition)*. Open Road Media.

Gottwald, Georg A. and Ian Melbourne (2004), "A new test for chaos in deterministic systems." *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 460(2042), 603–611.

Gottwald, Georg A and Ian Melbourne (2005), "Testing for chaos in deterministic systems with noise." *Physica D: Nonlinear Phenomena*, 212(1), 100–110.

Gradojevic, Nikola and Ramazan Gencay (2008), "Overnight interest rates and aggregate market expectations." *Economics Letters*, 100(1), 27–30.

Grassberger, Peter and Itamar Procaccia (1983a), "Characterization of strange attractors." *Physical Review Letters*, 50(5), 346.

Grassberger, Peter and Itamar Procaccia (1983b), "Estimation of the Kolmogorov entropy from a chaotic signal." *Physical review A*, 28(4), 2591.

Grassberger, Peter and Itamar Procaccia (1983c), "Measuring the strangeness of strange attractors." *Physica D: Nonlinear Phenomena*, 9(1), 189 – 208.

Harrison, Mary Ann and Ying-Cheng Lai (1999), "Route to high-dimensional chaos." *Physical Review E*, 59(4), R3799.

Ikeguchi, Tohru and Kazuyuki Aihara (1997), "Lyapunov spectral analysis on random data." *International Journal of Bifurcation and Chaos*, 7(6), 1267–1282.

Kaplan, Daniel T and Leon Glass (1992), "Direct test for determinism in a time series." *Physical Review Letters*, 68(4), 427.

Kodba, Stane, Matjaž Perc, and Marko Marhl (2004), "Detecting chaos from a time series." *European Journal of Physics*, 26(1), 205.

Kolmogorov, Andrei Nikolaevich (1958), "A new metric invariant of transient dynamical systems and automorphisms in Lebesgue spaces." In *Dokl. Akad. Nauk SSSR (NS)*, volume 119, 861–864.

Kyrtsou, Catherine, Anastasios G Malliaris, and Apostolos Serletis (2009), "Energy sector pricing: On the role of neglected nonlinearity." *Energy Economics*, 31(3), 492–502.

Li, Tien-Yien and James A Yorke (1975), "Period three implies chaos." *The American Mathematical Monthly*, 82(10), 985–992.

Lorenz, Edward N (1963), "Deterministic nonperiodic flow." *Journal of the Atmospheric Sciences*, 20(2), 130–141.

Martelli, Mario, Mai Dang, and Tanya Seph (1998), "Defining chaos." *Mathematics Magazine*, 71(2), 112–122.

Matilla-Garca, Mariano (2007), "Nonlinear dynamics in energy futures." *The Energy Journal*, 28(3), 7–29.

Ott, Edward (2002), *Chaos in dynamical systems*. Cambridge university press.

Panas, Epaminondas and Vassilia Ninni (2000), “Are oil markets chaotic? A non-linear dynamic analysis.” *Energy Economics*, 22(5), 549–568.

Rosenstein, Michael T, James J Collins, and Carlo J De Luca (1993), “A practical method for calculating largest Lyapunov exponents from small data sets.” *Physica D: Nonlinear Phenomena*, 65(1), 117–134.

Ruelle, David (1989), *Chaotic evolution and strange attractors*, volume 1. Cambridge University Press.

Saeed Moshiri, Faezeh Foroutan (2006), “Forecasting nonlinear crude oil futures prices.” *The Energy Journal*, 27(4), 81–95.

Schuster, Heinz Georg and Wolfram Just (2006), *Deterministic chaos: an introduction*. John Wiley & Sons.

Sinaĭ, Ja (1959), “On the concept of entropy for a dynamic system.” In *Dokl. Akad. Nauk SSSR*, volume 124, 768–771.

Strogatz, Steven H (2014), *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. Westview press.

Tanaka, Toshiyuki, Kazuyuki Aihara, and Masao Taki (1996), “Lyapunov exponents of random time series.” *Physical Review E*, 54(2), 2122.

Tanaka, Toshiyuki, Kazuyuki Aihara, and Masao Taki (1998), “Analysis of positive Lyapunov exponents from random time series.” *Physica D: Nonlinear Phenomena*, 111(1), 42–50.

Touhey, Pat (1997), “Yet another definition of chaos.” *The American Mathematical Monthly*, 104(5), 411–414.

Werndl, Charlotte (2009), “What are the new implications of chaos for unpredictability?” *The British Journal for the Philosophy of Science*, 60(1), 195–220.

Wiggins, Stephen (2013), *Chaotic transport in dynamical systems*, volume 2. Springer Science & Business Media.

Wolf, Alan, Jack B Swift, Harry L Swinney, and John A Vastano (1985), “Determining Lyapunov exponents from a time series.” *Physica D: Nonlinear Phenomena*, 16(3), 285–317.

Yousefpoor, P, MS Esfahani, and H Nojumi (2008), “Looking for systematic approach to select chaos tests.” *Applied Mathematics and Computation*, 198(1), 73–91.