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A SUGGESTION FOR A MULTIVARIATE CONCORDANCE COEFFICIENT

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DIPARTIMENTO DI ECONOMIA

A SUGGESTION FOR A MULTIVARIATE CONCORDANCE COEFFICIENT

Silvia Terzi - Luca Moroni

A suggestion for a multivariate concordance coefficient

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Abstract

In the present paper we will introduce a coefficient of multivariate association i.e. association in a $d-$variate vector of observations $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_d)$, where $d \geq 2$ and where each $\mathbf{x}_j$ is itself a vector of n observations.
We order the observations, divide them in slices and count how many times one observation in the $r-$th slice of any of the d distributions also belongs to the $r-$th slice of any of the others. The greater the number of overlaps between the units belonging to corresponding slices, the greater the concordance between the d distributions.
This is the simple and intuitive idea our multivariate association coefficient κ stems from. It is in fact a multidimensional concordance coefficient since it assumes comonotonicity for all variables.

Keywords: Copula, Concordance, Local concordance, Measures of multivariate association
JEL Classification: C140

1. Introduction

In the present paper we will introduce a coefficient of multivariate association i.e. association in a $d-$variate vector of observations $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_d)$, where $d \geq 2$ and where each $\mathbf{x}_j$ is itself a vector of n observations.
Quoting Nelsen (An introduction to copulas, p.157) *"Dependence properties and measures of association are interrelated, and so there are many places where we could begin this study. Because the most widely known scale-invariant measures of association are the population versions of Kendalls tau and Spearmans rho, both of which measure a form of dependence known as concordance, we will begin there."*
Scarsini (1984) and other authors (for example Joe 1997, Taylor 2007) give an axiomatic definition of a concordance measure κ, suitable for continuous bivariate variables and for $d-$variate generalizations. These measures are based on copulas[2] (Sklar, 1959) and lead to *concordance orderings* (or other types of *multivariate positive dependence orderings*) of **continuous** cumulative distribution functions (*cdf*) (see Decancq, 2010).

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[1]The authors would like to thank Prof. M.M.Barbieri for reviewing the paper.
[2] *"For continuous multivariate distributions, the univariate marginals and the multivariate - or dependence - structure can be separated; the dependence structure can be represented by a copula. The copula is a multivariate distribution with all univariate margins being continuous uniform distributions $U(0, 1)$."* (Joe, 1997, p.12).

Here instead we will not restrict attention to continuous variables, we will assume $d \geq 2$ and will pay attention only to the observations rather than to the random variables they have been sampled from; we aim at defining a concordance coefficient rather than concordance orderings.
Loosely speaking d variables are concordant if for a unit i ($i = 1, \ldots, n$) large values on some variables are associated with large values on all the others and conversely for another unit i small values for some variables are associated with small values on all the others. Perhaps the most widely used coefficient of concordance between 3 or more distributions is Kendalls W, designed to assess the agreement between d raters. Kendall's W ranges from 0 (no agreement) to 1 (complete agreement/greatest concordance).
We take moves from a similar concept of concordance/agreement. However, instead of ranking all the observations (as we would do for Kendalls W), we divide the d distributions in slices (windows) of s consecutive ordered observations and count how many times one observation in the $r-$th window of any of the d distributions also belongs to $r-$th window of any of the others. If the distributions are concordant the units associated to the observations belonging to the $r-$th slice of the $h-$th distribution will more or less coincide with the units associated to the observations of the $k-$th distribution ($\forall h, k = 1, \ldots, d$). The greater the number of overlaps/intersections between units of corresponding slices, the higher the concordance between the d distributions.
The outline of the paper is as follows. In §2 we introduce first of all a local concordance coefficient (and the notation required) and by means of sliding windows its global extension. In §3 we discuss some computational matters and in §4 we illustrate an example application measuring concordance between the pillars on which Global Competitiveness Index is based.

2. Concordance coefficient κ_r

Let $\mathbf{X}$ be the data matrix of n units and d variables: $\mathbf{X} = (\mathbf{x}_{i,h}), i = 1, \ldots, n, h = 1, \ldots, d$. Let $\mathcal{N}$ be a set of n uniquely identified (labelled) units. Consider the d rankings derived from the variables; and the integer[3] $s = \lfloor \frac{n}{d} \rfloor$ (we will discuss this position shortly below).
Let us divide each empirical distribution $\mathbf{x}_h$ in slices (windows) by means of ordered observations: each slice will refer to s consecutive ordered observations. Let $\mathcal{N}_r^h$ be the subset of s labels of the observations belonging to $r-$th slice of $h-$th distribution. First of all we define a local concordance coefficient.
Accordingly with Dolati - Úbeda-Flores (2006) description of multivariate concordance, *"Concordance of random variables means that they tend to be all large together or all small together"*, if the variables are concordant, we expect the subsets $\mathcal{N}_r^h, k = 1, \ldots, d$ to overlap to some extent, and more so as the concordance is more pronounced . In particular we expect the number of non empty intersections between subsets to increase. The maximum local concordance, i.e. the maximum concordance within a specific window, is reached when all the subsets overlap perfectly.
Vice-versa if the variables are not concordant, we expect the subsets to be more and more disjoint, to the point to be totally disjoint; this because of the marked difference between the positions of each unit on different rankings.
The choice of $s = \lfloor \frac{n}{d} \rfloor$ is crucial, as s is precisely the integer such that $ds \leq n$ while $d(s+1) > n$. If ds were greater than n, some subsets would necessarily overlap, whatever the concordance; it would be difficult, if ever possible, to distinguish between artificially induced and concordance induced overlaps. Conceptually, the optimal situation is when $ds = n$, as the subsets, in extreme

[3] By $\lfloor \cdot \rfloor$ we mean the integer part of the argument.

conditions of non-concordance, are a partition of the original sample: the subsets will be totally disjoint, but all the units of the sample will be involved. Choosing $n^* = ds$, the nearest available lower neighbor of n, induces the slightest conceptual bias, because the $n - ds$ units that will always be excluded from the subsets are the fewest possible, while avoiding any artificially induced overlap. This is a valuable feature, as we favour an assessment of the concordance over the evaluation of a more ambiguous notion of non-concordance.
Now, for a given window (for given r) consider the union of the subsets across the d dimensions, $\mathcal{N}_r = \bigcup_{h=1}^{d} \mathcal{N}_r^h$, and the cardinality[4] of the set $\mathcal{N}_r$, $C_r = |\mathcal{N}_r|$. C_r is the number of units that occupy a position in the r-th window on at least one of the d rankings. From the considerations above follows that, the greater the local concordance the greater the cardinality of the intersections between the $\mathcal{N}_r^h$; however in virtue of De Morgans law[5] the higher the cardinality of the intersections the lower the cardinality of the union, C_r. On the contrary very feeble local concordance (or no local concordance) will lead towards empty intersections and greater C_r. When local concordance is at its maximum, the $\mathcal{N}_r^h$ are identical, thus C_r reaches its minimum s; when local concordance is at its minimum, the $\mathcal{N}_r^h$ are totally disjoint, and thus $C_r = ds$, is maximum. This means that C_r is an inverse [absolute] measure of local concordance. A linear transformation of C_r gives the relative index of local concordance κ_r:

$$\kappa_r = \frac{ds - C_r}{ds - s} \tag{1}$$

and $0 \leq \kappa_r \leq 1$.
To define a global concordance index we compute a local concordance index κ_r for each sliding window $W_r = (r, \ldots, r + s - 1)$ of s consecutive ranks/integers (with $r \in \{1, \ldots, n - s + 1\}$), and then consider their average κ:

$$\kappa = \frac{1}{n - s + 1} \sum_{r=1}^{n-s+1} \kappa_r \tag{2}$$

Also $0 \leq \kappa \leq 1$.
κ reaches its superior limit if and only if each unit has exactly the same ranking over all the d dimensions; this is the case in which the d distributions show perfect concordance. As an intuitive proof of the sufficiency of this condition, suppose that one unit, on two rankings, falls in two different positions; then there exist at least one reference window that comprises only one of the two positions, and this in turn prevents the two related subsets to perfectly overlap, the local index to be 1 and the average to reach its maximum Vice-versa $\kappa = 0$ implies that the gaps/distances between the positions of each unit on the d rankings are at least equal to s, otherwise for some reference window some unit would be element of more than one subset.
Due to its local-to-global nature, κ is slightly affected by a boundary problem: the local concordance at the very extreme positions has marginally less influence on the global index than it has at the intermediate positions. There are a number of ways to try to circumvent this undesirable behavior, like weighting, restricting or extending (toroidally) the average. However while on one hand the resulting index would require a totally different framework to be interpreted, on the other the boundary effect on κ in most situations is so small to be irrelevant.

[4] The cardinality of a set A (denoted as $|A|$) is the number of its element.

[5] Let A_i be a collection of subsets of A, $i \in I$; De Morgan's law states that $\overline{\bigcap_{i \in I} A_i} \equiv \bigcup_{i \in I} \overline{A_i}$ and $\overline{\bigcup_{i \in I} A_i} \equiv \bigcap_{i \in I} \overline{A_i}$

3. Computational matters

To further illustrate the method and detail a possible computational pattern, a simple example from simulated data is presented below.
The example refer to a survey of 12 households on which 3 variables are observed: total net income (*income*), household's apartment current value (*value*), per-capita apartment square meters (*meters*). Let us start with a table containing the data (Table 1).

Table 1: Data

name	*income*	*value*	*meters*
Smith	33.7	373	26.7
Johnson	59.8	582	42.7
Williams	32.7	355	35.6
Brown	33.2	331	19.2
Jones	31.1	316	21.7
Miller	30.3	364	22.5
Davis	39.6	454	25.5
Garcia	40.2	540	33.8
Rodriguez	40.5	522	40.3
Wilson	54.5	607	34.7
Martinez	52.9	410	32.3
Anderson	54.6	560	38.6

Now consider the rankings of the households with respect to the three variables (Table 2).

Table 2: Rankings

rank	*income*	*value*	*meters*
1	Johnson	Wilson	Johnson
2	Anderson	Johnson	Rodriguez
3	Wilson	Anderson	Anderson
4	Martinez	Garcia	Williams
5	Rodriguez	Rodriguez	Wilson
6	Garcia	Davis	Garcia
7	Davis	Martinez	Martinez
8	Smith	Smith	Smith
9	Brown	Miller	Davis
10	William	Williams	Miller
11	Jones	Brown	Jones
12	Miller	Jones	Brown

Start to build the windows. In this example there are $n = 12$ units and $d = 3$ variables; the windows depth is thus $s = \frac{n}{d} = 4$. The 3 first window's sets (ranks 1 to 4) are:

$\mathcal{N}_1^{income}$ = {Johnson, Anderson, Wilson, Martinez}
$\mathcal{N}_1^{value}$ = {Wilson, Johnson, Anderson, Garcia}
$\mathcal{N}_1^{meters}$ = {Johnson, Rodriguez, Anderson, Williams}

Their union is:

$\mathcal{N}_1$ = {Johnson, Anderson, Wilson, Martinez, Garcia, Rodriguez, Williams}

The cardinality of $\mathcal{N}_1$ is:

$C_1 = |\mathcal{N}_1| = 7$

For the second windows (ranks 2 to 5) we have:

$\mathcal{N}_2^{income}$ = {Anderson, Wilson, Martinez, Rodriguez}
$\mathcal{N}_2^{value}$ = {Johnson, Anderson, Garcia, Rodriguez}
$\mathcal{N}_2^{meters}$ = {Rodriguez, Anderson, Williams, Wilson}

$\mathcal{N}_2$ = {Anderson, Wilson, Martinez, Rodriguez, Johnson, Garcia, Williams}

$C_2 = |\mathcal{N}_2| = 7$

Iterating the process, we obtain the vector of the C_r:

$$\mathbf{C} = (7, 7, 6, 6, 7, 6, 6, 7, 6)$$

Through normalization we obtain the local concordance indexes κ_r, shown in table 3.

Table 3: Local concordance indexes

κ_1	0.625
κ_2	0.625
κ_3	0.75
κ_4	0.75
κ_5	0.625
κ_6	0.75
κ_7	0.75
κ_8	0.625
κ_9	0.75

To analyse the local behavior of the concordance, we can plot the κ_r coefficients, as show in Figure 1.

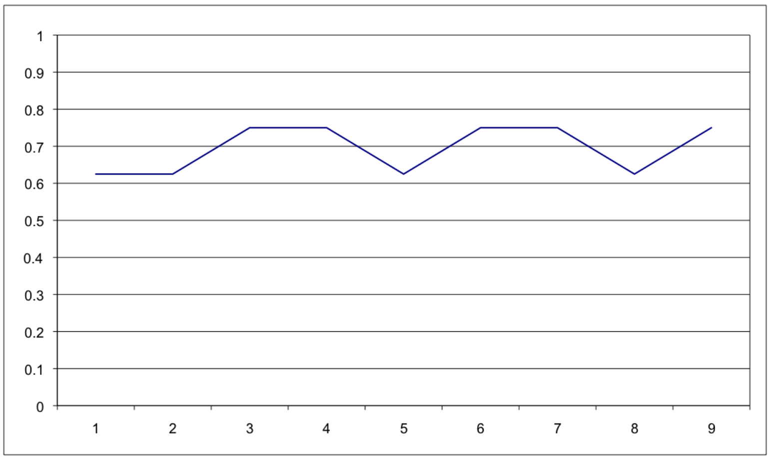


Figure 1: Local concordance

Finally, the average of the κ_r is the global concordance coefficient $\kappa = 0.6944$.

For larger datasets, a computational procedure can be based on presence-absence matrices. Consider the table (Table 4) that for each household lists its three ranking positions.

Table 4: Positions

name	*income*	*value*	*meters*
Smith	8	9	8
Johnson	1	2	1
Williams	10	6	4
Brown	9	11	12
Jones	11	12	11
Miller	12	10	10
Davis	7	7	9
Garcia	6	4	6
Rodriguez	5	5	2
Wilson	3	1	5
Martinez	4	8	7
Anderson	2	3	3

From it we derive the presence-absence matrices $\mathbf{P}^h, h = 1, 2, 3$; the rows refer to the units

$i = 1, \ldots, n$, and the columns to their ranks $j = 1, \ldots, n$. The generic element p^h_{ij} is 1 if the observation $x_i(h)$ referring to the unit i has rank j, 0 otherwise.
For each of the $n - s + 1 = 9$ reference windows, we consider a column vector $\mathbf{W}_r$ of dimension n such that its i−th element is equal to 1 if the unit i belongs to the r−th window and 0 if it doesnt. Each vector $\mathbf{W}_r$ has s adjacent 1s, while the other components are 0s.
Now define a matrix $\mathbf{W} = [\mathbf{W}_1, \ldots, \mathbf{W}_r, \ldots, \mathbf{W}_{n-s+1}]$ whose columns are given by the vectors $\mathbf{W}_r$:

$$\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrix product $\mathbf{P}^h \cdot \mathbf{W}$ gives a matrix, $\mathbf{L}^h$, whose columns are the $\mathcal{N}^h_r$ presence-absence vectors, e.g. for $\mathbf{L}^1$:

$$\mathbf{L}^1 = \mathbf{P}^1 \cdot \mathbf{W} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{W} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The next step consists in the computation of the matrix $\mathbf{L}$, whose columns are the presence-absence vectors of the $\mathcal{N}_r$ sets. It is obtained as the logical sum of the d $\mathbf{L}^h$ matrices; in other words, let l^h_{ij} be the generic element of $\mathbf{L}^h$, let l_{ij} be element of $\mathbf{L}$, then $l_{ij} = 0$ if and only if $l^h_{ij} = 0 \ \forall h = 1, \ldots, d$, and is 1 otherwise. The sum $\mathbf{C}$ of the row vectors of $\mathbf{L}$, a row vector itself, contains the C_r.

4. An application to the Global Competitiveness Index

The Global Competitiveness Report studies the many factors that favor national competitiveness. The principal tool of the analysis is the Global Competitiveness Index (GCI), a composite indicator based on 12 main determinants (pillars) each defined as an average of many different components; it measures the microeconomic an macroeconomic foundations of national competitiveness. A natural question could be: *how strong is the association between different components or between the different pillars? Could a strong association between some pillars lead to*

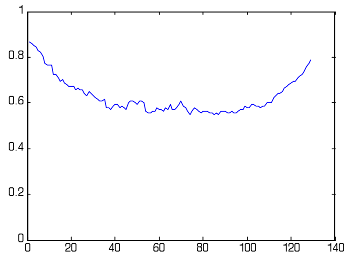
the definition of sub-indexes?
We attempt to answer some of these questions by means of our example application. First of all we computed the association between the 12 pillars of competitiveness.
The pillars are the following:

Pillar 1: Institutions
Pillar 2: Infrastructures
Pillar 3: Macroeconomic Environment
Pillar 4: Health and Primary Education
Pillar 5: Higher education and training
Pillar 6: Goods and market efficiency
Pillar 7: Labor market efficiency
Pillar 8: Financial market development
Pillar 9: Technological readiness
Pillar 10: Market size
Pillar 11: Business sophistication
Pillar 12: Innovation
(see appendix A for the exact composition of each pillar).

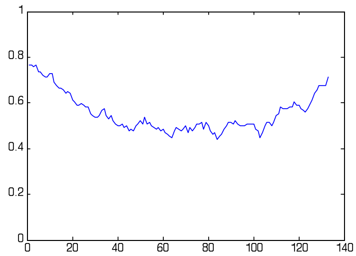
Being $n = 144$ and $d = 12$ we set $s = 12$ and obtained $\kappa = 0.5542$.
Next we gave a closer insight at the first 4 pillars. According to the Global Competitiveness Report the first 4 pillars (referring to institutions, infrastructure, macroeconomic environment and health and primary education) could give rise to a *basic requirement* sub index. How strong is their concordance? We obtained (setting $s = 36$) $\kappa = 0.5909$.
Similarly pillars from 5 to 10 give rise to a *efficiency enhancers* subindex. Their concordance is $\kappa = 0.5424$.
Pillars 11 and 12 give rise to *sophistication and sophistication factors* and concordance coefficient $\kappa = 0.8137$.
And how strong is the association within a pillar? In pillar 11, *Business sophistication*, we have 10 factors:

Local supplier quantity
Local supplier quality
State of cluster development
Nature of competitive advantage
Value chain breadth
Control of international distribution
Production Process sophistication
Extent of marketing
Willingness to delegate authority
Reliance on professional management

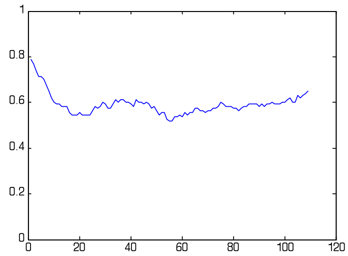
and $\kappa = 0.6283$.



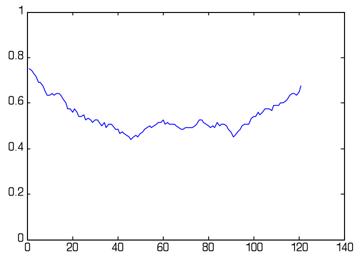
a: 11th pillar indicators, κ=0.6283



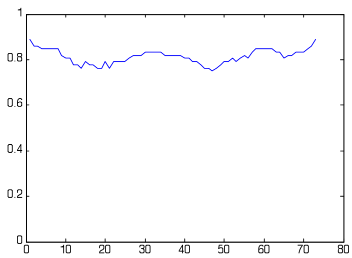
b: pillars 1-12, κ=0.5542



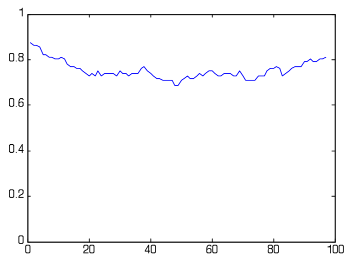
c: pillars 1-4, κ=0.5909



d: pillars 5-10, κ=0.5424



e: pillars 11-12, κ=0.8137



f: subindexes A, B and C, κ=0.7537

Figure 2: Local and global concordance coefficients for some of the data of GCI report 2012-2013

Appendix A. Description of GCI pillars and indicators

The pillars and indicators defined in the global competitiveness report are the following:

1. Institutions
A. Public institutions
1.01 Property rights
1.02 Intellectual property protection
1.03 Diversion of public funds
1.04 Public trust in politicians
1.05 Irregular payments and bribes
1.06 Judicial independence
1.07 Favoritism in decisions of government officials
1.08 Wastefulness of government spending
1.09 Burden of government regulation
1.10 Efficiency of legal framework in settling disputes
1.11 Efficiency of legal framework in challenging regulations
1.12 Transparency of government policymaking
1.13 Provision of government services for improved business performance
1.14 Business costs of terrorism
1.15 Business costs of crime and violence
1.16 Organized crime
1.17 Reliability of police services
B. Private institutions
1.18 Ethical behavior of firms
1.19 Strength of auditing and reporting standards
1.20 Efficacy of corporate boards
1.21 Protection of minority shareholders interests
1.22 Strength of investor protection

2.Infrastructure
A. Transport infrastructure
2.1 Quality of overall infrastructure
2.2 Quality of roads
2.3 Quality of railroad infrastructure e
2.4 Quality of port infrastructure
2.5 Quality of air transport infrastructure
2.6 Available airline seat kilometers
B. Electricity and telephony infrastructure
2.7 Quality of electricity supply
2.8 Mobile telephone subscriptions
2.9 Fixed telephone lines

3. Macroeconomic environment
3.1 Government budget balance
3.2 Gross national savings
3.3 Inflation
3.4 Government debt
3.5 Country credit rating

4. Health and primary education
A. Health
4.1 Business impact of malaria
4.2 Malaria incidence
4.3 Business impact of tuberculosis
4.4 Tuberculosis incidence
4.5 Business impact of HIV/AIDS
4.6 HIV prevalence
4.7 Infant mortality
4.8 Life expectancy
B. Primary education
4.9 Quality of primary education
4.10 Primary education enrollment rate

5.Higher education and training
A. Quantity of education
5.1 Secondary education enrollment rate
5.2 Tertiary education enrollment rate
B. Quality of education
5.3 Quality of the educational system
5.4 Quality of math and science education
5.5 Quality of management schools
5.6 Internet access in schools
C. On-the-job training
5.7 Local availability of specialized research and training services
5.8 Extent of staff training

6. Goods market efficiency
A. Competition
6.1 Intensity of local competition
6.2 Extent of market dominance
6.3 Effectiveness of anti-monopoly policy
6.4 Extent and effect of taxation
6.5 Total tax rate
6.6 Number of procedures required to start a business
6.7 Time required to start a business
6.8 Agricultural policy costs
6.9 Prevalence of trade barriers
6.10 Trade tariffs
6.11 Prevalence of foreign ownership
6.12 Business impact of rules on FDI
6.13 Burden of customs procedures
6.14 Imports as a percentage of GDP
B. Quality of demand conditions
6.15 Degree of customer orientation
6.16 Buyer sophistication

7. Labor market efficiency
A. Flexibility
7.1 Cooperation in labor-employer relations
7.2 Flexibility of wage determination
7.3 Hiring and firing practices
7.4 Redundancy costs
6.4 Extent and effect of taxation
B. Efficient use of talent
7.5 Pay and productivity
7.6 Reliance on professional management

7.7 Brain drain
7.8 Female participation in labor force

8. Financial market development
A. Efficiency
8.1 Availability of financial services
8.2 Affordability of financial services
8.3 Financing through local equity market
8.4 Ease of access to loans
8.5 Venture capital availability
B. Trustworthiness and confidence
8.6 Soundness of banks
8.7 Regulation of securities exchanges
8.8 Legal rights index

9. Technological readiness
A. Technological adoption
9.1 Availability of latest technologies
9.2 Firm-level technology absorption
9.3 FDI and technology transfer
B. ICT use
9.4 Internet users
9.5 Broadband Internet subscriptions
9.6 Internet bandwidth
9.7 Mobile broadband subscriptions
2.8 Mobile telephone subscriptions
2.9 Fixed telephone lines

10.Market size
A. Domestic market size
10.1 Domestic market size index
B. Foreign market size
10.2 Foreign market size index

11. Business sophistication
11.1 Local supplier quantity
11.2 Local supplier quality
11.3 State of cluster development
11.4 Nature of competitive advantage
11.5 Value chain breadth
11.6 Control of international distribution
11.7 Production process sophistication
11.8 Extent of marketing
11.9 Willingness to delegate authority
7.6 Reliance on professional management

12. R and D Innovation
12.1 Capacity for innovation
12.2 Quality of scientific research institutions
12.3 Company spending on R and D
12.4 University-industry collaboration in R and D
12.5 Government procurement of advanced technology products
12.6 Availability of scientists and engineers
12.7 PCT patent applications
1.02 Intellectual property protection

References

[1] Decancq, K.,"Copula-based orderings of multivariate dependence," CORE Discussion Papers 2010012, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE) (2010).
[2] Dolati A., Úbeda-Flores M., On Measures of Multivariate Concordance, Journal of Probability and Statistical Science 4(2), 147-163 (2006).
[3] Joe, H., Multivariate Models and Dependence Concepts, Chapman & Hall, London (1997).
[4] Nelsen, R.B., An introduction to copulas, 2nd ed., Springer, New York (2006).
[5] Scarsini, M., On measures of concordance, Stochastica 8, 201-218 (1984).
[6] Sklar,M., Fonctions de répartition à n dimensions et leurs marges, Publications de l'Institute de Statistique de l'Université de Paris, 8, 229-231 (1959).
[7] Taylor, M.D., Multivariate measures of concordance, Ann. Inst. Stat. Math. 59(4), 789-806 (2007).