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SOCIAL INCOME**

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**REDAZIONE:**

Dipartimento di Economia  
Università degli Studi Roma Tre  
Via Silvio D'Amico, 77 - 00145 Roma  
Tel. 0039-06-57335655 fax 0039-06-57335771  
E-mail: dip\_eco@uniroma3.it



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SOCIAL INCOME**

Attilio Trezzini

*Comitato Scientifico:*

*Fabrizio De Filippis*

*Anna Giunta*

*Paolo Lazzara*

*Loretta Mastroeni*

*Silvia Terzi*

## The Meaning of the Long-Run Ratio of Saving to Social Income

Attilio Trezzini\*  
Università di Roma Tre

### Abstract

*The ratio of saving to social income is generally conceived as the result of stable patterns of individual and institutional decisions to save. In a theoretical context in which aggregate demand is recognized as playing a part in the growth process positing a general assumption on consumption, it is possible to argue, instead, that the ratio of saving to income is also strongly affected by the incentive to invest. It is further argued, however, that without the assumption of steady-state conditions, the ratio of saving to income cannot be conceived as a magnitude in a precise relationship to the rate of accumulation or to any other single specific phenomenon.*

**Jel Codes:** E120, E210, O400

**Key Words:** Saving ratio, Economic Growth, Cyclical Fluctuations and Trends

### Introduction

This paper addresses the meaning of the *ratio* of saving to social income, which is generally conceived as the result of stable patterns of individual and institutional decisions to save. As such, it is also known as the *saving rate* or *average propensity to save*. The term “propensity” means the result of more or less rational decisions on the part of households and institutions about the allocation of income between saving and consumption.

It will be argued that this meaning cannot be attached to the ratio of saving to income. On the basis of an interpretation of the empirical literature on consumption, it is assumed that aggregate consumption depends partly on current income and partly on a set of other circumstances largely independent of current income. There is no assumption of an automatic tendency towards full employment in the theoretical context adopted, and aggregate demand is therefore recognized as playing a part in the growth process.

As in the literature that attributes aggregate demand with such a crucial role, it is argued that the ratio of saving to income is affected by the patterns of individual and institutional decisions to save but also and strongly by the rate of growth of aggregate demand.

It is further argued that without the assumption of steady-state conditions, the ratio of saving to income cannot be conceived as a magnitude in a precise relationship to the rate of

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accumulation or to any other single specific phenomenon. The value of this ratio is generally the result of the interaction of many different phenomena and can therefore be similar in radically different circumstances and different in similar circumstances with respect both to the incentive to accumulate and to saving-decision patterns.

### **The empirical evidence and theoretical insights into aggregate consumption**

Theoretical considerations on the meaning of the proportion of saving out of aggregate income cannot be separated from general reflection on the theoretical insights into the determinants of consumption and saving.

The first assumption about *aggregate* consumption, which was stated by Keynes in the *General Theory*, was simply based on “*our knowledge of human nature and [on] ...the detailed facts of experience*” (Keynes, 1936, p. 96).

Keynes developed a complex analysis of the determinants of consumption expenditure made up of heterogeneous principles.<sup>1</sup> It was however reduced by Keynes himself, and even more so by subsequent Keynesian analyses, to the element essential to the statement of the principle of effective demand. This essential element was the so-called “fundamental psychological law”: “men are disposed [...] to increase their consumption as their income increases, but not by as much as the increase in their income.” (Keynes, 1936, p. 96) Consumption is an increasing function of current income with a marginal propensity to consume lower than unity. To this fundamental law, Keynes added the assumption that the average propensity to consume decreases as income rises. This obviously implies the existence of a component of consumption that is largely independent of current income.

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<sup>1</sup>In Keynes’s analysis of consumption it is possible to discern an echo of the ideas constituting the approach of the social significance of consumption (see Trezzini (2010) and (2012)). “The subjective factors [...] include those psychological characteristics of human nature and those social practices and institutions which, though not unalterable, are unlikely to undergo a material change over a short period of time except in abnormal or revolutionary circumstances.” (Keynes, 1936, p. 91) Given that the marginal propensity to consume is lower than unity, “[t]his is especially the case of the so-called cyclical fluctuations of employment during which habits, as distinct from more permanent psychological propensities, are not given time enough to adapt themselves to changed objective circumstances. For a man’s habitual standard of life usually has the first claim on his income, and he is apt to save the difference which discovers itself between his actual income and the expense of his habitual standard; or, if he does adjust his expenditure to changes in his income, he will over short periods do so imperfectly.” (Keynes, 1936, p. 97). These passages refer both to the relevance of habits in determining consumption and to the residual nature of savings, two constituent principles of the approach based on the social significance of consumption. The relevance of these elements is, however, explicitly limited to short-period fluctuations during which the adjustment of expenditure to changes in income in accordance with “more permanent psychological propensities” can only take place imperfectly. These propensities seem to be interpreted as the expression of the more traditional principles also used by Keynes. It follows from this that the “Keynesian consumption function” assumed to represent Keynes’s analysis is an oversimplification. In this connection, see Cook (2000), who also argues that the success of this simplification was probably due to the apparently sound econometric results given by estimates based on this assumption at the beginning of the 1940s.

Since then, the literature has seen an alternation in focus between the part of consumption proportional to the level of and changes in current income and the components independent of current income.

The debate between Duesenberry, Friedman and Modigliani in the 1940s and '50s was based entirely on the interpretation of contrasting empirical results<sup>2</sup> and gave birth to the permanent income and life cycle hypotheses as the theoretical basis for most of the subsequent theoretical analyses of aggregate consumption.

The combination of the permanent income and life cycle hypotheses with the assumption of rational expectations (Hall 1978) gradually led to the assumption that “people use savings to smooth income fluctuations and that they should respond little if at all to changes in income that are anticipated. When this important theoretical prediction is violated, researchers conclude that consumption is excessively sensitive to anticipated income changes.” (Japelli and Pistaferri, 2010) In this theoretical context, the dependence of aggregate consumption on variations in current income pinpointed by empirical analyses is thus conceived and studied as *excessive sensitivity*.

The difficulties in explaining the origin of this excessive sensitivity have then led since the 1990s to the abandonment of aggregate analysis and to analyses based on microeconomic data, which do not, however, appear to have definitely solved the problem. In this more recent literature, excessive sensitivity is understood as the effect of different causes: binding liquidity constraints, non-separabilities between consumption and leisure, home production considerations, habit persistence, aggregation bias, and the durability of goods. Other ways of dealing with the problem have also been adopted, such as drawing distinctions between *expected* and *unexpected* income changes, between unexpected *permanent* and *transitory* shocks, between expected *decreases* and *increases* in income, and between expected *small* and *large* changes in income.<sup>3</sup>

Though far from a complete and detailed survey, this outline of the literature on consumption appears sufficient to identify a pattern of the evolution of aggregate consumption on which the literature as a whole converges. Aggregate consumption can be conceived as the *sum* of a component that depends (excessively or otherwise) on current income and a component that is independent of it and determined by the past evolution of consumption itself.

While this pattern is interpreted in most of the theoretical literature according to the principles of rational intertemporal allocation by rational individuals, it can also be read from a

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<sup>2</sup> See Thomas (1989) and Trezzini (2010)

<sup>3</sup> This outline of the more recent literature is again indebted to Japelli and Pistaferri (2010). See also Attanasio and Weber (2010)

radically different viewpoint. A theoretical approach to the determinants of aggregate consumption originally put forward by the American Institutionalists has been recently reconsidered,<sup>4</sup> and the primary principles deriving from this approach have been used in the context of a demand-driven growth analysis.<sup>5</sup> This theoretical approach could also lead to an interpretation of the described pattern of aggregate consumption whereby the part of consumption not directly dependent on current income is regarded as determined by the *acquired standard* of consumption of the society. The part of consumption that depends asymmetrically<sup>6</sup> on current income is instead regarded as the changes in consumption that, if sufficiently persistent, enable the acquired standard to evolve.

It is, however, important to stress that that arguments put forward here about the central question addressed do not depend on this specific *explanation* of this pattern of evolution of consumption. In other words, our conclusions hold whenever consumption is seen as the sum of two components, one directly dependent on current income and the other largely independent of it and connected to its past evolution, as it is in most empirical analyses.

### **The ratio of saving to aggregate income in the short-run**

Let us consider the problem in its simplest form, upon which it is possible to distinguish broad consensus, i.e. the logical framework generally called the *Keynesian short period*. This is characterized by the assumption of a stock of capital given in terms of level and composition as well as given money wages and prices.<sup>7</sup>

The following simplifying assumptions will apply throughout the paper. Investment, savings and the social product are gross. The economy is closed and there is no government intervention. Income at any time  $t$  is equal to the sum of consumption and investment:

$$Y_t = C_t + I_t \quad (1)$$

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<sup>4</sup> Trezzini (2005) and (2012).

<sup>5</sup> Garegnani and Trezzini (2010); Trezzini (2011).

<sup>6</sup> One of the basic principles of the Institutionalist theories based on the *social relevance* of consumption is the asymmetry of consumption expenditure. Expenditure on consumption proves more elastic with respect to positive than to negative changes in income. The marginal propensity to consume is thus assumed to be higher during periods of expansion than recessions.

<sup>7</sup> The meaning of these short-period assumptions in the General Theory has been widely debated. Many passages in *The General Theory* state and others seem to suggest that Keynes's real intent was to go beyond cyclical fluctuations and address the fundamental forces that tend to drive the determination of output and accumulation. The *General Theory* can thus be interpreted as laying the foundations for a long-period theory of output and accumulation. (In this sense see Eatwell and Milgate (1983) and Panico and Petri (1987)) There is no doubt, however, that the explicit assumption of a given level of fixed capital and other features of Keynes's criticism of traditional theories have proved conducive to its interpretation as a short-period theory of fluctuations. See Garegnani (1978-9).

Consumption is determined in accordance with the Keynesian aggregate consumption function:

$$C_t = \bar{C} + cY_t \quad (2)$$

In this short-run conceptual framework, the first component,  $\bar{C}$ , can be assumed as given and generically determined by the long-run evolution of habits. A given productive capacity is assumed and investment has a demand effect only. Under these assumptions, the equality of saving to investment is

$$S_t = Y_t(\bar{C} + cY_t) = I_t \quad (3)$$

And the ratio of saving to aggregate income is

$$\frac{S_t}{Y_t} = 1 - \frac{\bar{C}_t}{Y_t} - c \quad (4)$$

The effective demand principle implies that when the level of output changes, saving decisions adjust to actual investment. The inner logic of the Keynesian principle of effective demand implies that the proportion of income saved – in the short period – depends on the amount of investment to be accommodated. Higher investment generates higher income and thus a higher amount of savings, which adjusts to the autonomously determined investment. There is evidently no reason why the ratio of aggregate savings to aggregate income should be independent of the amount of investment.<sup>8</sup>

The value of the proportion of income saved changes as the fraction  $\bar{C}/Y_t$  – in relation (4) – changes. Given the level of  $\bar{C}$ , the fraction  $\bar{C}/Y_t$  is entirely determined by the actual level of output realized. Higher investment generates higher income and the increase in income generates a proportional increase of the “induced” part of consumption, but aggregate consumption grows less than proportionally. This allows a higher amount of saving that adjusts to the autonomously determined investment. A higher level of investment implies not only a higher level of saving but also a higher *ratio of saving to income*.

The short-period *marginal* propensity to consume measures the way in which consumption changes as income changes, and can be regarded as somehow reflecting individual or institutional saving behaviors. The value of the ratio of saving to income in each period is, however, not fully determined by such behavior.<sup>9</sup>

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<sup>8</sup> Under more general assumptions, exports and government expenditure would play a role similar to that of investment.

<sup>9</sup> It is important to stress that this is a logical result that does not change with the particular assumptions on the consumption function. The only case in which the proportion of income saved is completely determined by saving behaviors is that in which consumption is entirely proportional to disposable income and there are no components of demand independent of income other than investment. Only in this extreme case could the ratio of saving to income

This conclusion can be confirmed and developed for more general hypotheses than those defining the Keynesian short period.

### **A methodological premise**

Before proceeding, it is necessary to make it clear that once the well-defined area of Keynesian short-run analysis is abandoned, there is no explicit consensus about the method to be used in order to analyze the long-term evolution of produced quantities and productive capacity.

The distinction between short-period and long-period analysis was explicitly developed by Marshall<sup>10</sup> with a clear statement of the method generally accepted in the economic theory of value and distribution since the Classical economists. These ideas have been widely discussed in the analyses of prices and distribution, as have the changes that have taken place in the same concepts during the history of economic thought and their origins.<sup>11</sup> Albeit at the cost of crude simplification, it is possible to identify the assumptions about the level and composition of productive capacity as the criterion upon which short-period and long-period analyses are distinguished in analyses of prices and distribution. Capital stock and productive capacity are assumed as given in the short run, which makes it impossible to assume a uniform rate of return on capital. In long-run analyses, it is instead assumed that capital stock and productive capacity can change in level and composition so as to adjust to the conditions required for a uniform rate of return on capital. This is a condition characterizing long-period analyses.

It is impossible to find explicit definitions of short-run and long-run analyses when the object addressed is the evolution of produced quantities over time. Marginalist theories are essentially general equilibrium analyses and, as such, determine prices and produced quantities simultaneously. In this context, it is thus natural and logically necessary to extend the same concept of short-run and long-run analysis used in the theory of prices to the theory of quantities. This feature is peculiar to these theories, however, and further discussion and analysis is required in any other theoretical context.

Moreover, when the object of the analysis is the temporal evolution of produced quantities, the distinction between short-run and long-run analyses overlaps with another relevant distinction. Economies generally develop in time through fluctuations in the levels of produced output that describe a trend. The relations between the fluctuations of produced quantities and their trend levels may appear to be similar to the relations between short-run

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coincide with the marginal propensity to save, but it would ultimately be no less dependent on investment than in the other circumstances.

<sup>10</sup> Marshall (1890), especially book V, chapter V.

<sup>11</sup> Garegnani (1976)

prices and their fluctuations towards their long-run levels. But the relation between cycles and trend magnitudes has seldom been explicitly theorized in the literature.

What can be observed is a general tendency to study trends independently of fluctuations and cyclical fluctuations independently of trends.<sup>12</sup> In this way, the relation between long-period prices and the fluctuations of actual prices around them of the Classical and original marginalist analyses tends to be transplanted into the analysis of relations between cyclical fluctuations and trend magnitudes.

Since Solow's contribution in 1956, the trend of produced quantities has been studied in growth analyses by means of steady-state paths, conditions in which all the variables grow at the same rate. This feature is common to practically all the analyses of growth, not only those deriving from Solow's model and based on traditional theories but also those recognizing the relevance of aggregate demand in the growth process.

Discussion of this methodological issue lies beyond the scope of this work. Moreover, it is by no means easy to grasp the implications of abstract reasoning on method for the explanation of economic phenomena. Our analysis, then, addresses a value of the ratio of saving to income determined as the simple average of the values assumed by the same magnitudes in each single period of a cyclical fluctuation. This magnitude is considered relevant *in some way* to analysis of the long-run tendencies of economies.

While this cannot be regarded as putting forward an alternative method of investigation, which would require more detailed and explicit analysis, it appears sufficient to demonstrate the limitations of the method widely used in demand-driven growth models.

### **The theoretical context and some general assumptions**

The number of phenomena affecting the economy increases dramatically in the long run. Further simplifying assumptions are therefore needed in order to address our issue in such a complex framework. Technical progress is ignored and constant returns to scale are assumed with no scarce natural resources. Labor is indefinitely available. A given real wage and the corresponding system of competitive normal prices are assumed. The growth of the economy will therefore not affect relative prices.<sup>13</sup> As before, there is a closed economy with no

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<sup>12</sup> It is certainly possible to find relevant exceptions to this general tendency, which did not, however, succeed in imposing different methods. See Kaldor (1954) Kalecky (1968).

<sup>13</sup> The latter assumptions serve to distinguish the study of quantities from the study of prices and distribution, a feature deriving from the approach of the classical political economists. This does not, however, preclude the influence of changes in distribution on economic growth or vice versa and the ensuing changes in prices. These feedback relations can be studied separately.

government intervention in which the only components of aggregate demand are consumption and investment. Aggregate output, investment and saving are taken as gross.

The discussion is developed in a theoretical context where the level of economic activity and its growth are dependent on aggregate demand in both the long run and the short run. High elasticity of long-period output in relation to changes in aggregate demand is recognized and there is no assumption of an automatic long-run tendency towards full employment.

As stated above, the aggregate consumption  $C$  is regarded as the sum of what can be called a *structural component*<sup>14</sup> and a component proportional to aggregate income in accordance with a marginal propensity to consume  $c$ . The *structural* component can, however, no longer be assumed as given. It evolves period after period according to a dynamics that is independent of current income and generally determined by the past evolution of consumption, even though many other phenomena also affect this evolution. Most of the latter phenomena – such as product innovations, changes in income distribution, and institutional changes – are ruled out by simplifying assumptions but their influence is not theoretically excluded.

A full analysis of investment lies far beyond the scope of the present paper. Just two essential assumptions are required in order to develop our argument in its simplest terms. The first is a descriptive feature: investment fluctuates and generates fluctuations in output and income. Through cyclical fluctuations, investment thus follows an increasing, decreasing or constant trend. The second is that this trend is determined by the tendency of capacity to adjust to demand *more or less completely*, i.e. with no need to assume that the tendency is *realized* even on average.<sup>15</sup> The average trend of investment is in fact determined *independently* of the hypothetical trend of capacity saving, understood as saving corresponding to the income obtained by the continuous normal utilization of productive capacity.

### **Changes in the saving ratio over the business cycle**

Let us now consider how the ratio of saving to income – as described by relation [4] – changes during the business cycle.

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<sup>14</sup> The term “*autonomous* consumption” is deliberately not used here because it is ambiguous and essentially suitable for short-run analysis, in which this component of consumption can be taken as given independent of current income and not explained. It must instead be explained in a long-run context, and in our specific analysis the explanation of its evolution must be sought in the process of habit formation and the constant acquisition of changing standards of consumption. In this sense, this component of aggregate consumption cannot be considered autonomous with respect to income in a very general sense.

<sup>15</sup> A complete adjustment of capacity to demand implies the prevalence of normal utilization of productive capacity. It is not assumed in our analysis that this condition is obtained in the long-run. On this point, see Trezzini (1995), (1998) and Palumbo and Trezzini (2001).

$$\frac{S_t}{Y_t} = 1 - \frac{\bar{C}_t}{Y_t} - c \quad (4)$$

The level of investment increases at annual rates  $\mathbf{g}_{It}$  during expansion, which determines increases in income according to the multiplier. Consumption changes as well. Part of aggregate consumption grows proportionally to income according to the marginal propensity  $c$  and the other part according to the dynamics of the *structural* component, which changes at annual rates  $\mathbf{g}_{ct}$ .

As the values of  $\mathbf{g}_{ct}$  are determined autonomously, they can be regarded as differing from the growth rates both of investment,  $\mathbf{g}_{It}$ , and of aggregate income, which is in turn determined by the multiplier and the expansion of the sum of structural consumption and investment.

In order to simplify, let us assume that the rates  $\mathbf{g}_{It}$  are constant and equal to  $\mathbf{g}_I^e$  during expansion and that the rates  $\mathbf{g}_{ct}$  also are constant during expansion and equal to  $\mathbf{g}_c^e$ . It can also be assumed at first that structural consumption grows at a lower rate with respect to investment, and the condition  $\mathbf{g}_c^e < \mathbf{g}_y^e < \mathbf{g}_I^e$  will therefore follow.

The multiplicative process always generates saving corresponding to investment. During expansion, the proportion of *consumption* in income decreases as the trend component of consumption increases according to a growth rate lower than that of income. On the other hand, the proportion of investment in income – and thus of saving in income – increases during the adjustment of saving to investment.

Investment decreases after a peak period and this generates a recession. For simplicity, let us again assume a constant rate of change in investment during the recession,  $\mathbf{g}_I^r$ , which is negative by assumption. Aggregate consumption changes because part of it is proportional to income and part follows its structural dynamics. The constant rate of change of the structural component of consumption during recession,  $\mathbf{g}_c^r$ , can in principle be anything. What emerges as the most plausible assumption from unbiased observation of the behavior of aggregate consumption during recessions, however, is that if this component does not continue to increase, it remains constant or at least decreases at a lower rate than investment. As a result, it is probable that  $\mathbf{g}_c^r > \mathbf{g}_y^r > \mathbf{g}_I^r$ .

The proportion of *consumption* in income thus increases during recessions. The proportion of investment – and hence of saving – in income decreases during the adjustment of saving to investment.

In this simple economy, the ratio of saving to income is equal to that of investment in income and it therefore fluctuates during the cycle, being determined in each period by the amount of investment to be accommodated. There is clearly no reason for the average proportion

of income saved to be determined independently of fluctuations in investment or of the average level *and share* of investment to be accommodated.

### The determination of an average value of the saving ratio over the business cycle

Let us now use relation [4] to understand the changes in the ratio of saving to income over the business cycles. A simple transformation leads to the following relation:

$$\frac{S_t}{Y_t} = \frac{(1-c)}{\frac{C_t}{I_t} + 1} \quad (5)$$

The saving ratio is determined in each period by the ratio of structural consumption  $\bar{C}_t$  to aggregate investment  $I_t$ , neither of which can now be regarded as constant. Any statement about the long-run value of the saving rate must be based on analysis of the evolution of the ratio  $\bar{C}_t/I_t$  over time.

Let us assume that the oscillations in investment take place in a cycle of duration  $T$  composed of  $\theta$  periods of expansion and  $\tau$  periods of recession. Different rates of change must be considered. While investment varies in each period of expansion at a rate  $g_I^e$ , which is taken as constant for simplicity, it declines during recessions at a rate  $g_I^r$ , which is again constant for simplicity. Similar assumptions can be made for the structural component of consumption, which is assumed to grow at a rate  $g_c^e$  during expansion and to change at a rate  $g_c^r$  (which can in principle be negative, zero or positive) during recession.

During expansion, for each  $t = 1, \dots, \theta$ , the saving ratio is:

$$\frac{S_t}{Y_t} = \frac{(1-c)}{\frac{\bar{C}_0(1+g_c^e)^t}{I_0(1+g_I^e)^t} + 1} \quad (6)$$

where  $\bar{C}_0/I_0$  is the value of the last period of the previous cycle. This ratio reaches its maximum value in the peak period  $t = \theta$ , when the fraction  $\bar{C}_\theta/I_\theta$  reaches its minimum. During recession periods,  $t = \theta+1, \dots, \tau$ , the saving ratio is determined as:

$$\frac{S_t}{Y_t} = \frac{(1-c)}{\frac{\bar{C}_0(1+g_c^e)^\theta(1+g_c^r)^{t-\theta}}{I_0(1+g_I^e)^\theta(1+g_I^r)^{t-\theta}} + 1} \quad [7]$$

The average value of the ratio of saving to income over the  $T = \theta + \tau$  periods of the business cycle can be expressed as:

$$\left(\frac{S}{Y}\right)_{av} = \frac{(1-c)}{\theta + \tau} \left\{ \left[ \sum_{i=1}^{\theta} \frac{1}{\frac{\bar{C}_0}{I_0} (1+g_c^e)^i + 1} \right] + \left[ \sum_{j=1}^{\tau} \frac{1}{\frac{\bar{C}_0}{I_0} (1+g_c^e)^{\theta} (1+g_c^r)^j + 1} \right] \right\} \quad [8]$$

The analysis of relation [8], under our simplified assumptions, makes it possible to draw some general conclusions, which will be compared in the next section with those to be found in the literature.

The average value of the ratio of saving to aggregate income over the business cycle is the joint result of a *set of circumstances*.

Some of these can be connected with a generic *notion of thrift*, i.e. to the behavior of households and institutions with respect to the allocation of income between consumption and saving. In particular, the average value of the saving ratio over the business cycle is *inversely* proportional to the marginal propensity to consume  $c$  and *inversely* proportional to the rates of growth of the structural component of consumption during expansion and recession  $g_c^e$  and  $g_c^r$ . In this sense, the average rate of saving appears to be higher the less intense the dynamic of aggregate consumption or, in more general terms, the less favorable to consumption the behavior of households and institutions.

The ratio of saving to income cannot, however, be understood as determined exclusively by the saving behavior of households and institutions, since it is also affected by other circumstances of a different nature. As such, the ratio should therefore not be seen as a *propensity*.

Another set of circumstances affecting the average ratio of saving to income over the cycle is connected with what can be called the *incentive to invest*. From relation (8) it is possible to argue that given the marginal propensity to consume and the evolution of the structural component of consumption, this average ratio is *directly* proportional to rates of growth of investment both in expansion and in recession,  $g_I^e$  and  $g_I^r$ .

As briefly considered below, the pattern of evolution of investment is generally summarized in the literature by the average rate of accumulation or by the average rate of growth of investment  $\bar{g}_I$ . In relation [8] this influence of the average rate of growth of investment is obviously implicit.<sup>16</sup>

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<sup>16</sup> In our argument, this average rate can take any value. Investments are autonomous magnitudes and no constraining assumptions are made as regards their patterns of fluctuation. Their average value  $\bar{g}_I$  is an autonomous variable as well.

It is, however, important to stress a second conclusion of our analysis, namely that what affects the saving ratio is not only the incentive to invest as represented by the average rate of growth of investment, but also the particular ways in which this incentive manifests itself.

The same average rate of growth of investment over the whole business cycle can emerge from fluctuations that differ in intensity and duration. These intensities and durations are not irrelevant in determining the ratio of saving to income. Given the rates  $g_I^e$  and  $g_I^r$ , the average ratio of saving to income is directly proportional to  $\theta$  and inversely proportional to  $\tau$ . In this way, the *whole pattern of fluctuations* in investment can be seen to affect that average value.

In appendix one, after definition of the average values of the growth rates of investment and structural consumption, relation [8] is also expressed as a function of these average values. This makes it possible to show our conclusions, which can be stated as follows. i) The average saving ratio is determined by the magnitudes expressing saving/consumption behaviors and by other circumstances. ii) Among these, the average growth rate of investment  $\bar{g}_I$  has a direct effect on this ratio. iii) The same average growth rate of investment  $\bar{g}_I$  can, however, lead to different average ratios of saving to income depending on whether they result from expansions and recessions of greater or lesser intensity or from longer expansions and shorter recessions. All the magnitudes  $g_I^e$ ,  $g_I^r$ ,  $g_c^e$ ,  $g_c^r$ ,  $\theta$  and  $\tau$  are relevant in determining the average value of the saving ratio.

In this way, even when the behavior of households and institutions with respect to saving and consumption is kept constant, it is not possible to state a univocal relation between the average growth rate of investment and the average ratio of saving to income.

The same average value of the saving to income ratio can result from conditions differing radically with respect not only to saving behaviors but also to the intensity of the process of growth and accumulation. It can be concluded from this that as the result of interaction between many different phenomena, the average value of the ratio of saving to income can be similar in radically different circumstances and different in similar circumstances with respect to both saving and investment behaviors.

### **The ratio of saving to output under less restrictive assumptions**

The above considerations hold under the strict assumptions used to obtain equation [8], some of which are particularly “heroic”. It is assumed that all the variables are mutually independent, and this is obviously not so in general. From the standpoint of a demand-led growth approach, it would be plausible to assume a dependence of the growth rates of investment and of

(structural) consumption. In this case, any increase in  $g_c^e$  could prompt an increase in  $g_r^e$ . The net effect on the average value of the saving ratio would depend on the relative change in the two rates. An increase in the two rates such as to leave the relation  $1+g_c^e/1+g_r^e$  unchanged would be the only condition in which the average value could remain unaffected. In general, however, the relative change in the two rates is another determinant of the average rate of saving.

Symmetrical cases can be conceived with respect to changes in the two rates during recessions. Moreover, it is not possible to rule out the existence of cross-dependency of the different rates of growth of the variables. The dynamics of investment or consumption during expansion would in principle affect the dynamics of investment and/or consumption during recession. The duration of the phases would in fact affect the way in which consumption changes<sup>17</sup>. The analysis of all these possible cases would obviously make our reasoning far more complicated without, however, leading to any general conclusions independent of the specific assumption used in the model. Moreover, all this hypothetical analysis would in no way impair our primary conclusions, which would rather be strengthened.

Furthermore, the number of circumstances affecting the average value of the saving ratio increases dramatically as soon as our simplifying assumptions are relaxed and other components of aggregate demand are taken into consideration. Exports or government expenditure in both gross and “net” terms affect the determination of the average value of the saving to income ratio in a far more complex way than the simple dynamics of structural consumption with respect to investment does in our simplified case. The conclusion that the ratio of saving to income represents neither the propensity of an economy to save nor any other individual phenomena nor simple interrelations between a few phenomena seems to be even stronger in this more general case than in the simplified one.

There is at least one other point to be discussed, namely the assumption of constant *long-run income distribution*, which has two kinds of implications on our reasoning as regards the rate of saving.

This assumption makes it possible to separate the analysis of the determinants of growth and accumulation from the analysis of prices and distribution. This implies that the primary determinants of the process of development can be identified even independently of changes in distribution but not that changes in distribution are neutral with respect to the process of

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<sup>17</sup> The assumption that the structural component of consumption is determined by the acquisition of socially determined standards of consumption would entail the view that a longer period of expansion would have a stronger effect on the consolidation of habits than a short one. The length of the expansion would thus affect the rate of change of consumption during the recession.

accumulation. Changes in income distribution may affect aggregate demand through changes in the marginal propensity to consume or the dynamics of structural consumption or through changes in the incentive to invest. In this sense, income distribution should be regarded as affecting nearly all the variables that determine the long-run saving rate in the form of the average value of that magnitude over cyclical fluctuations.

There is, however, another point connected with income distribution: the assumption of given *normal* distribution (and prices) does not imply that actual incomes, the only ones of relevance for actual saving, can be considered constant during cyclical fluctuations.

It is plausible that capacity utilization tends to be higher than normal during expansion, which implies actual aggregate profits higher than those corresponding to normal utilization, whereas wages can instead be more in line with their normal levels despite this buoyancy. It is plausible that profits tend to decrease during recessions with higher elasticity than wages. When a recession is sharp, however, unemployment may lead to relevant contractions in aggregate actual wages. The way in which fluctuations affect actual incomes even in situations of constant normal distribution appears to be another complex issue that affects the average value of the saving ratio. This influence seems to depend on many different features of the fluctuations.

### **The long-run *propensity* to save in the literature of full employment**

Comparison of our conclusions with the literature makes it possible to explain them more clearly. First and foremost, they are radically at variance with the conclusions of the mainstream literature, which sees the ratio of saving to aggregate income *in the long run* as univocally determined by individual and institutional saving behaviors. As stated, it is because this specific meaning is attributed to this magnitude that it is generally called the *propensity* to save.

This complex set of theories explicitly or implicitly identifies long-run conditions with the full employment of productive resources. In the original formulations of this position, the tendency towards full employment is argued on the basis of regularly decreasing demand functions for productive factors (and goods) and the assumption of flexibility of rates of remuneration (and prices of goods).

Solow regards the condition of full employment as a prerequisite for the analysis of long-run phenomena and economic growth.<sup>18</sup> In his analysis the assumption of full employment is used in a dynamic framework analogous to that suggested by Harrod (1939). This leads to a view

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<sup>18</sup> “..Once we know the time path of capital stock and that of the labor force, we can compute from the production function the corresponding time path of real output. .... Since the real return to factors will adjust to bring about full employment of labor and capital we can use the production function ... to find the current rate of output. Then the propensity to save tells us how much of net output will be saved and invested. “Solow (1956) p.68

of steady-state paths as the positions to which economies tend. Solow states the tendency of capital stock to adjust to the trend required by the continuous full employment of the workforce, which grows in number due to demographic factors and in productive potential due to technological progress. Since Solow's contribution, steady-state paths, characterized by a homogeneous rate of growth of income and capital stock, which also implies equal rates of growth for all the components of aggregate demand and supply became the center of the theories of growth. Steady-state assumptions continue to characterize the entire corpus of mainstream theories of growth despite the many and relevant changes introduced by more recent formulations, such as endogenous growth models. These assumptions imply that the rate of growth of investment is not an autonomous magnitude. The same principles leading to the tendency towards full employment imply the tendency of investment always to adjust to the level of saving corresponding to the full employment of income. It is thus the evolution of saving that determines the evolution of investment.

In the corpus of the mainstream theories, another set of shared principles and assumptions is used to state the tendency towards a constant long-run value of the saving ratio. As outlined above, the idea of aggregate consumption made up of a structural component and a component proportional to current income – which is essentially common to all the empirical analyses – is seen as the result of individual processes of intertemporal maximization of utility leading to rational allocations of incomes between consumption and saving. Above and beyond cyclical fluctuations, individuals tend to carry out “consumption smoothing” and thus arrive at an optimal allocation of income in accordance with constant proportions of income saved and consumed.

Given the tendency towards full employment, steady-state conditions and consumption smoothing, our reasoning as regards relation [8] obviously cannot hold. The average value of the actual saving ratios over the business cycle is not particularly relevant, as these values constantly tend to coincide with the long-run values determined by the theory. The average value may also differ from the theoretical long-run value, but this difference can be regarded as determined by accidental factors and hence negligible.

Even though the relation between actual and theoretical magnitudes – and their reciprocal correspondence – is both never explicitly discussed and highly questionable in principle, the existence of long-run steady-state values seems to be a strong basis, in this context, for the study of growth independent of cyclical fluctuations and a view of actual magnitudes as expressions of the forces studied by the theory.

### **The proportion of saving in demand-led growth analyses**

Our conclusions have been reached within a theoretical framework that differs radically from the one in which full employment is assumed as a feature defining long-run conditions. It is therefore particularly important to compare them with those reached by analyses that assume a similar theoretical context, i.e. those that seek to extend the Keynesian principle of the relevance of aggregate demand to the examination of growth.

All these analyses agree with ours on one relevant point but diverge from it partially on others. The common origin of the differences is to be sought in the way in which the long-run tendencies of the economy are studied. These tendencies are generally studied by means of paths conceived as independent of the cyclical fluctuations through which the process of growth actually takes place.

Some analyses use steady-state paths along which all the magnitudes grow at the same rate and which are associated with conditions of normal utilisation of productive capacity. Some instead determine the trend of the economy in the form of a steady-state path characterized by a degree of capacity utilization that is endogenously determined and different from the normal. Others study economic growth by determining a path of balanced growth characterized by normal utilization, which is also assumed as the trend around which cyclical fluctuations take place, a trend which is, however, determined independently of cyclical fluctuations. Together with specific assumptions on consumption behavior, these assumptions about the trend of the economy lead in each approach to a unique value of the ratio of saving to income being identified out of the endless possible average values implied by relation [8] as the only one of relevance to the analysis of growth.

In the analyses based on the Cambridge equation,<sup>19</sup> an autonomously determined rate of capital accumulation determines a corresponding steady-state rate of growth of the economy. A simplified assumption is posited on the relation between consumption and income: no structural components are assumed and consumption is proportional to income. Saving behavior is identified by the different (average) marginal propensities to save of different social classes. The assumption of steady-state conditions, associated in this case with normal utilization, implies the complete rigidity of long-run output with respect to changes in aggregate demand. This rigidity in turn implies that the rate of accumulation determines income distribution and thus the prevailing aggregate ratio of saving to income. A general and regular correspondence between the rate of growth and the saving ratio is stated in this case: the higher the rate of accumulation, the higher the rate of saving.

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<sup>19</sup> Among the many works following this approach, Kaldor, N. (1956) and Robinson, J. (1956) (1962) can be taken as points of reference.

A different approach can be found in the neo-Kaleckian models.<sup>20</sup> On the basis of a critique of the Cambridge equation, these analyses see an autonomously determined rate of accumulation as determining a corresponding amount of saving in time by means of changes in the degree of capacity utilization associated with a steady-state trend. In particular, changes in realized profits make possible the changes in saving needed in order to adjust to investment.

The assumption of a steady state, associated here with non-normal capacity utilization, is essential in order to single out a unique value of the saving ratio in these models too. Only two factors are relevant in determining the saving ratio: the saving behaviors, i.e. the propensities to save on profits and on wages and the rate of accumulation (and growth). In these models too, given saving behaviors, it is possible to state a *direct* and univocal relation between the rate of growth and accumulation and the saving ratio.

Finally, we must recall the analyses based on the *supermultiplier*.<sup>21</sup> These analyses assign a key role to the expansion of aggregate demand and no role to changes in distribution in the adjustment of saving to investment. In this respect, supermultiplier analyses are the closest to our approach. They also assume a component of consumption proportional to current income and a structural component of this aggregate, and assign a determining role in the growth process precisely to the evolution of this structural component.

Economic growth is studied here by means of a trend characterized by a normal degree of capacity utilization. This assumption implies that all the magnitudes grow at the same constant rate, determined in this case by the rate of growth of autonomous demand (our structural consumption). This trend is seen not as a steady-state path but as an average trend resulting from fluctuations in the actual levels of the magnitudes. It is, however, determined independently of cyclical fluctuations.<sup>22</sup> Assuming a long-run trend characterized by a normal degree of utilization and independent of fluctuations is tantamount to substituting  $\mathbf{g}_c^e$  and  $\mathbf{g}_c^r$  with just one  $\mathbf{g}_c$  and  $\mathbf{g}_I^e$  and  $\mathbf{g}_I^r$  with just one  $\mathbf{g}_I$  in relation [8] and finally assuming that  $\mathbf{g}_c$  and  $\mathbf{g}_I$  coincide.

Our relation [8] thus collapses into

$$\left(\frac{S}{Y}\right)_{LR} = \frac{(1-c)}{\frac{C}{I} + 1} \quad (9)$$

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<sup>20</sup> This view was originally put forward in Rowthorn (1981). Other works taking the same view include Amadeo (1986) Dutt, A.K. (1990) Lavoie M. (1992) (1995).

<sup>21</sup> The major points of reference for this view are Serrano (1995), Bortis (1997) Cesaratto S., Serrano F. and Stirati A. (2003).

<sup>22</sup> All the properties of these models of growth are made explicit in the critical discussion of them in Trezzini (1995) and (1998).

where the average value over any time interval coincides with a long-run value relative to each individual period.

In these analyses, investment is necessarily only induced and also equal on average to the level of corresponding capacity savings. Its rate of growth is assumed to be equal to, and determined by, that of “autonomous consumption”. In these theoretical analyses the uniform rate of growth is assumed to be determined by an autonomously determined rate of growth of autonomous demand (in our simplified assumption the average rate  $g_c$ ). Adding the condition of normal utilization, the initial value of the fraction  $\bar{C}/I$  is univocally determined and remains constant all the way along the long-period path.

In this way the average/long-run saving ratio in relation (9) is entirely determined, as in the other Keynesian approach, by saving behavior (in particular  $c$  and the rate of growth of  $g_c$ ) and by the methodological condition of defining the trend path (either steady state or normal utilization or both). Here too, given the propensity to consume and the uniform rate of growth, it is possible to assign only one value to the ratio of saving to income.

### **Concluding remarks**

Comparison of our conclusions with those implicit in the literature shows that they depend on two primary characteristics of our analysis.

First of all, there is no assumption of any automatic tendency of economies towards full-employment equilibria. Our first result has to do with the theory of output and in particular with the assumptions about the role of demand in the process of growth. When this role is acknowledged, the *incentive to invest* – understood as the growth rates of investment during the process of accumulation – must be regarded as a force determining the ratio of saving to income.

Our analysis shows in fact that all the rates  $g_I^e$ ,  $g_I^f$  and their average value affect the average ratio of saving to income over business fluctuations.

Traditional analyses rule out this role of investment growth by virtue of the very principles on the basis of which the tendency towards full employment is assumed. These principles also imply a continuous tendency of investment to be determined by full-employment saving.

In stating a role of the intensity of the accumulation in determining the ratio of saving to income, our conclusions are *similar* to those reached by all the analyses that recognize the role of the expansion of aggregate demand in the growth process and hence also the role of the rate of accumulation/investment in determining the saving ratio.

Another characteristic of our analysis, and one that plays a crucial role in reaching our conclusions, is methodological in character. Our attention has focused on the simple average, over the business cycle, of the values that the ratio of saving to income actually assumes in each period of it. This value has been assumed as relevant for analysis of the long-run tendencies of economies.

In this way, it is assumed that long-run tendencies manifest themselves through fluctuations and thus determine the average/long-run values of magnitudes. Under this assumption, longer or more intense expansions and/or shorter and less intense recessions are the only ways through which a higher long-run rate of growth of investment (or income) manifests itself.

This marks a departure from what is implicitly assumed in the literature, where the long-run tendencies of the economy are determined independently of the features of cyclical fluctuations. Long-run tendencies are assumed to exist prior to the fluctuations, which are then necessarily regarded as random disturbances around long-run values.

The latter approach is strongly rooted in the theories of prices and distribution. But the extension of the same concept of short-run and long-run analysis to the theory of quantities is natural and logically necessary in the marginalist approach only. Marginalist theories are inherently general equilibrium analyses and, as such, determine prices and produced quantities simultaneously. This feature is, however, peculiar to these theories and further discussion and analysis is required in any other theoretical context.<sup>23</sup>

Our second set of conclusions has then a great deal to do with the method used to study long-run tendencies. The relevance of the pattern of change in investment is not determined only by the average value of its rates of growth, and it is misleading to study long-run tendencies by means of trends of balanced growth determined independently of fluctuations. This method precludes by definition all the circumstances connected with differences in the patterns of change of different components of aggregate demand or those connected with different intensities and duration of the cyclical phases.

These circumstances, which appear to be far more numerous than the handful shown in our highly simplified examples, prevent us from assigning any general and univocal meaning to the ratio of saving to income, just as it prevents us from suggesting any univocal relation between the rate of growth and the ratio of saving to income. All these circumstances must be regarded as manifestations of the autonomous role of the components of aggregate demand in the growth process.

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<sup>23</sup> This is particularly true when the theoretical approach assumed is that of the classical economists, where the analysis of prices and distribution is developed in a different logical stage from the analysis of produced quantities. See Garegnani (1984).

This method of analysis, though widely used in the theories studying the role of aggregate demand in the growth process, is not derived purely from the fundamental principles of these theories. It has been criticized from different angles, both as excessively simplistic and as deeply misleading. The present paper is intended as a further step towards overcoming the difficulty.

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### Appendix A

Let us define the average values of the growth rates of investment and the structural component of consumption over the business cycle as follows:

$$\bar{g}_I = \frac{\theta g_I^e + \tau g_I^r}{\theta + \tau} \quad \text{and} \quad \bar{g}_c = \frac{\theta g_c^e + \tau g_c^r}{\theta + \tau}$$

Let us then express one of each couple of rates as a function of the other and their average value:

$$g_c^e = [(\theta + \tau)\bar{g}_c - g_c^r] \quad g_I^e = [(\theta + \tau)\bar{g}_I - g_I^r]$$

By substituting  $g_c^e$  and  $g_I^e$  in relation [8] and rearranging, we obtain:

$$\left(\frac{S}{Y}\right)_{AV} = \frac{1-c}{\theta + \tau} \left\{ \sum_{i=1}^{\theta} \frac{\bar{c}_0/I_0 A \left\{ 1 + [(\theta + \tau)\bar{g}_c - g_c^r \tau] \left(\frac{1}{\theta}\right)^i + \left\{ 1 + [(\theta + \tau)\bar{g}_I - g_I^r \tau] \left(\frac{1}{\theta}\right)^i \right\}}{\bar{c}_0/I_0 \left\{ 1 + [(\theta + \tau)\bar{g}_c - g_c^r \tau] \left(\frac{1}{\theta}\right)^i \right\}} \right. \right. \\ \left. \left. + \sum_{j=1}^{\tau} \frac{\left\{ \left[ 1 + [(\theta + \tau)\bar{g}_I - g_I^r \tau] \left(\frac{1}{\theta}\right)^{\theta} (1 + g_I^r)^j \right\}}{\bar{c}_0/I_0 \left\{ 1 + [(\theta + \tau)\bar{g}_c - g_c^r \tau] \left(\frac{1}{\theta}\right)^{\theta} (1 + g_c^r)^j + \left\{ 1 + [(\theta + \tau)\bar{g}_I - g_I^r \tau] \left(\frac{1}{\theta}\right)^{\theta} (1 + g_I^r)^j \right\}} \right\}} \right\} \right.$$

As it is impossible to eliminate the influence of the whole pattern of economic fluctuations,  $\tau$ ,  $\theta$ ,  $g_I^r$  and  $g_c^r$ , from this relation, they are therefore determinants of the average value of the ratio of saving to income over the business cycle.

Let us now posit steady-state conditions:

$$g_c^e = g_c^r = \bar{g}_c = g_I^e = g_I^r = \bar{g}_I = H$$

These conditions are also implied by the less restrictive assumption that the economy automatically manages on average to attain a normal degree of utilization of productive capacity. By means of these conditions, we obtain:

$$\left(\frac{S}{Y}\right)_{AV} = \frac{1-c}{\theta+\tau} \left\{ \sum_{i=1}^{\theta} \frac{\bar{C}_0/I_0 \left\{ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right\}^i + \left\{ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right\}^i}{\bar{C}_0/I_0 \left\{ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right\}^i} + \sum_{j=1}^{\tau} \frac{\left\{ \left[ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right]^{\theta} (1+H)^j \right\}}{\bar{C}_0/I_0 \left\{ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right\}^{\theta} (1+H)^j + \left\{ 1 + [(\theta+\tau)H - H\tau] \left(\frac{1}{\theta}\right) \right\}^{\theta} (1+H)} \right\}$$

And through further simplification:

$$\left(\frac{S}{Y}\right)_{av} = \frac{(1-c)}{\bar{C}_0/I_0 + 1}$$