ROMA TRE
UNIVERSITÀ DEGLI STUDI DI ROMA TRE
DIPARTIMENTO DI ECONOMIA

CREDIT FRICTIONS, HOUSING PRICES AND OPTIMAL MONETARY POLICY RULES

Caterina Mendicino, Andrea Pescatori

Working paper n. 42
2004

ARACNE

1 Introduction

The recent rise in housing prices in most of the OECD countries has attracted the attention of policy makers and academics and raised concerns about the macroeconomic implications[1].

A number of papers have tried to understand the extend to which asset prices movements should be relevant for monetary policy[2]. Cecchetti et al. (2000 and 2002) show that reacting to asset prices, reduce the likelihood of bubbles forming. On the other hand, Bernanke and Gertler (2001), among others, conclude that inflation-targeting central banks should not respond to asset prices. In fact, conditional on a strong response to inflation, the gain from responding to asset prices is negligible. Both studies employ a financial accelerator framework allowing for credit market frictions and exogenous asset prices bubbles. The methodology adopted for evaluating the performance of different monetary policy rules is based on the implied volatility of output and inflation. Different conclusions about the desiderability of including asset prices as an additional argument in the monetary policy rule, depend on different assumptions about the stochastic nature of the model, i.e. the shocks considered.

Directly related to housing prices is the analysis by Iacoviello (2004). He shows the relevance of housing prices in the transmission and amplification of shocks to the real sector. Nevertheless, computing the inflation-output volatility frontiers turns out that a response to housing prices does not yield significant gains in terms of output and inflation stabilization.

The main shortcoming of all this literature is the absence of welfare considerations in evaluating optimal monetary policy. Only exception is the analysis conducted by Faia and Monacelli (2004). Relying on a welfare-based approach they show that reactions to asset prices do not generate relevant welfare improvements. More interesting, responses to changes in the leverage ratio generate more pronounced deviations from a strict price stability policy.

[1] See among others Borio and Mc Guire (2004) for the relation between housing and equity prices, Iacoviello (2004) for the relevance of housing prices and credit constraints in the business cycle, Girouard-Blöndal (2001) for the role of housing prices in sustaining consumption spending in the recent downturn of the world economy, Case-Quiugley-Shiller (2001) for empirical evidence on housing wealth effect.

[2] See e.g. Filardo (2000), Goodhart (2000), Batini and Nelson (2000), Bernanke and Gertler (1999, 2001), Cecchetti, Genberg, Lipsky and Wadhwani (2000), Cecchetti, Genberg and Wadhwani(2003), Taylor(2001), Kontonikas and Montagnoli (2003), Faia and Monacelli (2004).

This paper studies optimal monetary policy rules in an economy with credit market frictions at the household level and heterogeneous agents. The aim is to asses the role of household indebtedness and housing prices in designing monetary policy. The paper is related to the large literature on optimal monetary policy in economies with nominal rigidities[3]. This literature assumes that the central bank is a benevolent policy maker, thus, maximizes consumers' welfare[4]. Most of the models consider a dynamic system centered around an efficient non-distorted equilibrium. In practice, the policy maker neutralizes any source of inefficiency present in the economy and not related to the existence of nominal rigidities. Thus, the only duty left to monetary policy is to offset the distortions associated with price rigidities in order to replicate the flexible price equilibrium allocation. The motivation behind this modelling choices is purely technical. In fact, it is sufficient a first order approximation of the equilibrium conditions to approximate welfare up to the second order[5]. Following a method introduced by Rotemberg and Woodford (1997) in these kinds of models it is possible to derive a a discounted quadratic loss function from the quadratic approximation of the utility function, and compute optimal policy using a simple linear-quadratic methodology as in the traditional monetary policy theory.

An alternative approach, studies optimal monetary and fiscal policy in models evolving around equilibria that remain distorted[6]. These are models in which different types of distortions, beside price rigidities, proved a rationale for the conduct of monetary policy. In order to get a welfare measure that is accurate to the second order[7] it is necessary an higher order approximation of the model's equilibrium conditions. The method suggested by Schmitt-Grohe and Uribe (2003) shows that, a second order solution to the model's policy functions is required for the approximation of the welfare function be accurate up to the second order. Another way of evaluating a welfare measure accurate up to

[3] See among others, Rotemberg and Woodford (1997), Clarida, Gali and Gertler (1999), King and Wolman (1999), Erceg, Henderson and Levin (2000),

[4] The literature is divided in two streams on the base of a main assumption regarding the deterministic equilibrium around which the model economy evolves.

[5] See Woodford (2003)

[6] See Uribe and Schmitt-Grohe (2004), Benigno and Woodford (2004), Faia and Monacelli (2004).

[7] Up to a first order accuracy the agents' discounted utility function equals its non-stochastic steady state value. Since the monetary policy rules commonly considered do not affect the non-stochastic steady state, it is not possible to rank different rules on the base of first order approximation.

the second order, is proposed by Benigno and Woodford (2003) as an extension of Rotemberg and Woodford's method. On the base of the computation of a second order approximation to the model's structural equations it is possible to substitute out the linear terms in the Taylor approximation to the expected utility and obtain a "pure quadratic" approximation to the welfare function (no linear terms). Once a quadratic function is derived optimal monetary policy can be evaluated using as constraints the first order approximation to the model's equations. Thus, the linear-quadratic methodology is reintroduced again.

Our model economy is characterized by three types of distortions. First, nominal price rigidities, modelled as quadratic adjustment cost on good market price setting are adopted as a source of monetary non neutrality. Second, monopolistic competition in the good market allows for price setting above the marginal cost. Third, credit market imperfections, generate by the assumption that creditors cannot force debtors to repay unless debts are secured by collateral, generate a role for housing prices and household indebtedness.

Even though the relevance of liquidity constraints for consumption behavior has been well documented in the empirical and theoretical literature -see Zeldes (1997), Jappelli and Pagano (1997) among others- little attention has been given to credit frictions at the household level in the monetary business cycle literature. In fact, this paper represent the first attempt of a welfare-based monetary policy evaluation in a model with heterogeneous agents and credit constraints at the household level.

The model is built on Kiyotaki and Moore (1997) (KM henceforth). In order to generate a motive for the existence of credit flows, two types of agents are assumed. They differ in terms of discount factors: as a consequence impatient agents are borrowers. Credit constraints arise because lenders cannot force borrowers to repay. Thus, physical assets are used as collateral for loans. As in Iacoviello (2004), we depart from the framework from two main features. First, differently from KM we focus on the household's sector. In fact, KM's agents are entrepreneurs that produce and consume the same good using a physical asset. Agents are risk neutral and represent two different sectors of the economy - borrowers are "farmers" and lenders are "gatherers". On the contrary, we model households that, a part from getting utility from a flow of consumption and disutility from labor according to a standard concave function, and consider house holding as a separate argument of their utility function. Housing services are assumed to be proportional to the real amount of housing stock held. In our setup both groups of agents are identical, only difference is the subjective discount factor. Second, we extend the model to include nominal price rigidities and a role for monetary policy. In Iacoviello (2004) houses are not only a source of direct utility but also an input of production and the asset used in the credit market to secure both firm's and household's debts[8]. These modelling choices are consistent with the aim of showing the importance of financial factors for macroeconomic fluctuation. Instead, being interested in the role of housing prices and household debt leverage for the optimal design of monetary policy, we restrict our attention to the household sector. In order to evaluate optimal monetary policy we take advantage of the recent advances in computational economics by following the approach illustrated by Schmitt-Grohe and Uribe (2003).

As a result, an optimally designed simple monetary policy rule should not take into account deviations of the housing prices from the steady state level. However, the presence of credit frictions in the economy gives a role to the households' indebtedness level. In fact, taking into account the private sector indebtedness level, central banks are able to smooth better the effects of shocks to the aggregate economy.

The remainder of the paper is organized as follows. Section 2 describes the role of housing as a collateral. Section 3 lays out the model and derived the equilibrium conditions. Section 4 turns its attention on the model's calibration. Section 5 describes the welfare measure considered and the methodology to evaluate monetary policy's optimal design. Section 6 comments the results.

[8] Iacoviello (2004), as Faia and Monacelli (2004),adds collateral constraints tied to firms' real estate holdings (housing) to Bernanke, Gertler and Gilchrist (2000) model. Moreover, he also introduces collater constraits in the household sector.

2 Modelling Borrowing Constraint and Housing

Our main hypothesis is that housing is used as a collateral in the loan market and consequently housing prices are related to consumption and economic activity through both a traditional wealth effect and a mortgage loans market channel. We consider a modified version of the standard business cycle model in which household derive utility from owning houses and using them as collateral in the loan market. We depart from the representative agent framework assuming two groups of agents: borrowers and lenders.

Borrowers face an external borrowing constraint. The constraint is not derived endogenously but it is consistent with standard lending criteria used in the mortgage and consumer loans market. The borrowing constraint is introduced through the assumption that household cannot borrow more than a fraction of the value of their houses. The household borrows (B_{it}) against the value of his housing wealth.

$$B_{it} \leq \gamma E_t[Q_{t+1} h_{it}] \tag{1}$$

where Q_{t+1} is the housing prices and h_{it} is the stock of housing. Mortgage loans refinancing takes place every period and the household repays every new loan after one period. It seems quite realistic that the overall value of the loan cannot be higher than a fraction of the expected value of the collateral. The fraction γ, referred to as *loan to value ratio*, should not exceed one. This can be explained thinking of the overall judicial costs which a creditor incurs in case of the debtor default. Since housing prices affect the collateral value of the houses, fluctuations in the price plays a large role in the determination of borrowing conditions at household level. Borrowing against an higher value of the house is used to finance both investment in housing and consumption. The other source of mortgage equity withdrawal is given by an increase in the value of the collateral due to a rise the loan to value ratio.

3 The Model

Consider a sticky prices model populated by a monopolistic competitive good producing firm, a monetary authority and two types of households. In order to impose the existence of flows of credit in this economy we assume ex-ante heterogeneity at the household level: agents differs in terms of the subjective discount factor. We assume a continuum of households of mass 1: n *Impatient Households* (lower discount rate) that borrow in equilibrium and (1-n) *Patient Households* (higher discount rate) that lend in equilibrium.

3.1 Households

The households derive utility from a flow of consumption and services from house holding - that are assumed to be proportional to the real amount of housing stock held - and disutility from labor:

$$\max_{\{c_{it},h_{it},L_{it}\}} E\sum_{t=0}^{\infty}\beta_i^t U(c_{it},h_{it},L_{it})$$

with $i = 1, 2\text{and } \beta_1 > \beta_2$

s.t. a *budget constraint*

$$c_{it} + q_t(h_{it} - h_{it-1}) + \frac{b_{it-1}}{\pi_t} = \frac{b_{it}}{R_t} + w_t L_{it} + f_{it} - T_{it}$$

and a *borrowing constraint*

$$b_{it} \leq \gamma E_t[q_{t+1}\pi_{t+1}h_{it}] \tag{2}$$

All the variables are expressed in real terms, π_t is the gross inflation (P_t/P_{t-1}) and q_t is the price of housing in real terms (Q_t/P_t). The household can borrow (b_t) using as a collateral the next period's expected value of real estate holdings (the stock of housing). This borrowing constraint will hold only for the impatient households since the patient ones will lend in equilibrium. In the budget constraint T_{it} are lump sum taxes from the fiscal authority, and $f_{it}$are the dividends from firms.We assume that only the patient households own the firms. Thus, $f_{1t} = \frac{1}{(1-N)}(D_t/p_t)$ where D_t are the dividends of the representative firm and $f_{2t} = 0$.

3.1.1 Patient Households

Since the patient households' borrowing constraint is not binding in a neighborhood of the steady state he faces a standard problem, only exception is the existence of the housing services in the utility function:

labor supply

$$-U_{L_{1t}} = U_{c_{1t}} w_t$$

euler condition

$$U_{c_{1t}} = \beta_1 E_t^{-\varphi_c} U_{c_{1t+1}} \frac{R_t}{\pi_{t+1}}$$

housing demand

$$U_{h_{1t}} + \beta_1 E_t U_{c_{1t+1}} q_{t+1} = U_{c_{1t}} q_t$$

3.1.2 Impatient Households

Impatient Households borrow in a neighborhood of the steady state. We denote with μ_t the lagrange multiplier associated to the borrowing constraint.

labor supply

$$-U_{L_{2t}} = U_{c_{2t}} w_t$$

euler condition

$$\frac{U_{c_{2t}}}{R_t} - \mu_t = \beta_2 E_t U_{c_{2t+1}} \frac{1}{\pi_{t+1}}$$

housing demand

$$U_{h_{2t}} + \beta_2 E_t U_{c_{2t+1}} q_{t+1} + \mu_t \gamma E_t q_{t+1} \pi_{t+1} = U_{c_{2t}} q_t$$

3.2 Firms

3.2.1 The final good producing firms

Perfectly competitive firms produce a final good y_t using $y_t(i)$ units of each intermediate good $i \in (0,1)$ adopting a constant return to scale, diminishing marginal product and constant elasticity of substitution technology:

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

with $\theta > 1$. Costs minimization implies

$$\min_{\{y_t(i)\}} \int_0^1 P_t(i) y_t(i) di$$
$s.t.$the production function.

The price of the intermediate good $y_t(i)$ is denoted by $P_t(i)$ and taken as given by the competitive final good producing firm. The solution yields the following constant price elasticity (θ) demand function for good i that is homogeneous of degree one in the total final output:

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

Combining the demand function with the production function is possible to derive the price index for intermediate goods:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/(1-\theta)}$$

3.2.2 The intermediate sector

In the wholesale sector there is a continuum of firms indexed by $i \in (0,1)$ and owned by consumers. Intermediate producing firms act on a monopolistic market and produce $y_t(i)$ units of differentiated good i using $L_t(i)$ units of labor according to the following constant return to scale technology

$$Z_t L_t(i) \geq y_t(i)$$

where Z_t is the aggregate productivity shock and follows the autoregressive process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt}, \qquad \varepsilon_{Zt} \backsim^{iid} N(0, \sigma_{\varepsilon_Z}), \quad 0 < \rho_Z < 1$$

Cost Minimization Monopolistic competitive firms hire labor from households in a competitive market on period by period basis. Each firm chooses inputs minimizing total costs:

$$\min_{L_t(i)} [W_t L_t(i)]$$
$$s.t.$$
$$\bar{y}_t(i) = Z_t L_t(i)$$

Now we can find the nominal marginal cost s_t^n:

$$\frac{W_t}{Z_t} = s_t^n(i) \tag{7}$$

and thus the total cost could be written in the following way[9]:

[9] In equilibrium the firm chooses input such that the marginal product equals the markup times the factor price. In fact, in terms of gross markup $(1+\eta_t) = \frac{1}{s_t}$:

$$\frac{\bar{y}_t(i)}{L_t^*(i)} = (1+\eta_t)W_t$$

$$W_t L_t(i) = s_t^n(i) y_t(i)$$

Price Setting Assume now that intermediate firms set the price of their differentiated good every period, but facing a quadratic cost of adjusting the price between periods. The cost is measured in terms of the final good[10]:

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2$$

where $\phi_p > 0$ represent the degree of nominal rigidity and π is the gross steady state inflation. Each firm face the following problem:

$$\max_{\{P_t(i), L_t(i),\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left[\frac{D_t(i)}{P_t}\right]$$
$$s.t.$$
$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

Where $\Lambda_{t,t+j} = \beta_1^j \frac{U_{c1t+j}}{U_{c1t}}$ is the *relevant discount factor.* The firm's profits in real terms are given by :

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} y_t(i) - s_t(i) y_t(i) - \frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2$$

Using the results from the cost minimization problem, we can replace the real total costs , $w_t L_t(i)$,with a function of real marginal costs and total output[11].Thus, substituting for the total costs and the firm's production, the profits maximization problem becomes:

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left\{ y_t\left[\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} - s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\right] - \frac{\phi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2\right\}$$

The derivative with respect to the firm's price, multiplied for the price level P_t, yields:

$$0 = E_t \Lambda_{t,t+1}\left[\phi_p \frac{P_t}{\pi}\frac{P_{t+1}(i)}{P_t(i)^2}\left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right)\right] +$$
$$+ y_t\left[(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} + \theta s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}\right] - \phi_p \frac{P_t}{\pi P_{t-1}(i)}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)$$

[10] See Kim JME 1995

[11]
$$w_t L_t(i) = s_t(i) y_t(i) = \frac{w_t}{Z_t} y_t(i)$$

3.3 The Fiscal Authority

We assume:

$$G_t = T_t$$

where G_t is government consumption of the final good and T_t are lump sum taxes/transfers, where $T_t = (1-n)T_{1t} + nT_{2t}$.Government consumption evolves according to the following exogenous process:

$$(InG_t - InG) = \rho_G\,(InG_{t-1} - InG) + \varepsilon_{Gt} \quad \text{where} \quad \varepsilon_{Gt} \backsim^{iid} N(0, \sigma_{\varepsilon G}), \quad 0 < \rho_G < 1$$

where G is the steady state share of government consumption.

3.4 The Monetary Authority

We assume that the central bank follows an interest rate rule of the form:

$$R_t = \Theta(R_{t-1}, \frac{\pi_t}{\pi_{ss}})$$

According to which the nominal interest rate reacts to the contemporaneous deviation of inflation from it's (non-stochastic) steady state level and the lagged nominal interest rate.

3.5 Equilibrium and Aggregation

3.5.1 Equilibrium Conditions

In the symmetric equilibrium, all firms make identical decisions, so that:

$$y_t(i) = Y_t \qquad P_t(i) = P_t \qquad L(i) = L_t$$

Consequently, total production becomes:

$$Y_t = Z_t L_t \tag{E.4}$$

and the price setting equation:

$$0 = E_t U_{c1t+1}\left[\phi_p \frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\right] + U_{c1t}\left\{y_t\left[\theta\left(s_t - \frac{\theta-1}{\theta}\right)\right] - \phi_p \frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right)\right\}$$

Market Clearing conditions:

$$\begin{array}{cc} (1-n)L_{1t}+nL_{2t}=L_t & (1-n)c_{1t}+nc_{2t}=C_t \\ (1-n)b_{1t}+nb_{2t}=0 & (1-n)h_{1t}+nh_{2t}=1 \\ T_t=(1-n)T_{1t}+nT_{2t} & G_t=T_t \end{array}$$

Resource constraint:

$$Y_t = C_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2 + G_t \tag{E.10}$$

The production of the final sector needs to be allocated to resources costs arising from the prices' adjustment and to private consumption by households and government. These condition together with the household's and firm's first order conditions, the law of motion of the exogenous shocks, the central bank policy rule, the borrowing constraint and one of the two budget constraint constitute a system of non linear difference equations describing the behavior in equilibrium of prices ad quantities.After loglinearizing the system around its steady state we obtain the system of linear difference equations that determines the dynamics of the state and costate variables.

4 Calibration

We set the parameters of the model on the base of quarterly evidence. The household's discount factors are $(\beta_1, \beta_2) = (0.99, 0.98)$.Patient Households' discount factor implies an average annual rate of return of about 4%. Previous estimated of discount factors for poor or young households[12] as been used as a reference in the calibration of β_2. Regarding the utility function[13] we assume log-utility for consumption,$\varphi_c = 1$ (risk aversion), and we set $\varphi_L = 0.1$ (labor supply elasticity). The weight on labor disutility, ν_L,equals 1, while $\nu_h = 0.1$. This last parameter implies a steady state value of real estate over annual output of 1.03%. In line with the literature on nominal rigidities, we set the elasticity of substitution, θ, equal 11. The baseline choice for the loan to value ratio, γ, is 60% and the fraction of borrowed constraint population is settle to 50%. We calibrate the steady state government consumption value as the 20% of total

[12] In fact, Lawrance (1991) and Samwick (1998) estimate discount factors, respectively, for poor and young households in the range (0.97, 0.98).

[13] $U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L}$

output. Following Schmitt-Grohe and Uribe (2004) we calibrate the technology and government spending shocks according to standard values in the real business cycle literature[14]. Tab. 1 summarizes the calibrated parameters.

PREFERENCES		
$\beta_1 = 0.99$	$\varphi_c = 1$	$\nu_h = 0.1$
$\beta_2 = 0.98$	$\varphi_L = 0.1$	$\nu_L = 1$
TECHNOLOGY		
$\theta = 11$		
$\phi_p = 161$		
SHOCKS		
$\rho_Z = 0.95$	$\sigma_Z = 0.0074$	
$\rho_G = 0.9$	$\sigma_G = 0.0056$	

*Tab.*1

5 Computation and Welfare Measure

5.1 Computation

Since Kydland and Prescott (1982)[15] the first-order approximation approach is the most popular numerical approximation method for solving models too complex to deliver an exact solution. However, first order approximations may produce clearly erroneous results[16]. Comparing welfare among implementable policy rules that have no first-order effects on the model's deterministic steady state, we need to rely on higher order approximation methods.

As shown by Kim and Kim (2003)[17], in this context first order approximation methods are not locally accurate. In general a second-order accurate approximation to the welfare function requires a second-order expansion to the model's equilibrium conditions. First order approximation solution, is not always accurate enough due to the certainty equivalence property, i.e. the coincidence of the

[14] See, Cooley & Prescott (1995, chapter 1 in Cooley's book), or Prescott 1986.

[15] They applied to a real business cycle model a special case of the method of linear approximation around deterministic steady states developed in Magill (1977).

[16] See e.g. Tesar (1992) for an example where completing asset markets will make all agents worse off, Kim and Kim (2003) for stressing the same results in a two agents stochastic model.

[17] They show that a welfare comparison based on linear approximation to the policy functions of a simple two-countries economy, may yield the odd result that welfare is higher under autarky than under full risk-sharing.

first order approximation to the unconditional means of endogenous variables with their non stochastic steady state values. This neglects important effects of uncertainty on the average level of households' welfare. A first order approximation to the policy functions would give an incorrect second order approximation of the welfare function [18].

To overcome this limitation and obtain a second-order accurate approximation, we adopt a perturbation technique introduced by Fleming (1971) and applied to various types of economic models by Judd and coauthors[19] and recently generalized by Schmitt-Grohe and Uribe (2002)[20] (SU henceforth). Second order approximations are quite convenient to implement since, even capturing the effects of uncertainty, do not suffer from the "curse of dimensionality"[21].In fact, following SU, given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-orther terms can be identified by solving a linear system of equations whose terms are the first order terms and the derivatives up to the second order of the equilibrium conditions evaluated at the non-stochastic steady state.

5.2 Welfare Measure and Optimal Rules

How should monetary policy be conducted in a world economy with credit frictions at the household level? In order to answer this question, we rely on utility-based welfare calculations, assuming that the benevolent monetary authority maximize the utility of the households subject to the model's equilibrium conditions. Formally, the optimal policy maximize the household's life-time utility:

[18] See Woodford (2002) and Kim et al. (200?) for a discussion of situations in which second-order accurate welfare evaluations can be obtained using first-order approximations to the policy functions.

[19] See Judd and Guu (1993,1997) for applications to deterministic and stochastic, continuous and discrete-time growth models in one state variable, Gaspar and Judd (1997) for multidimentional stochastic models in continuous time approximated up to the fourth-order, Judd (1998) presents the general method , Jin and Judd (2001) extended these methods to more general rational expectations models .

[20] The derive a second-order approximation to the policy function of a general class of dynamic, discrete-time, rational expectations models. They show that in a second-order expansion of the policy functions, the coefficients on the linear and quadratic terms in the state vector are independent of the volatility of the exogenous shocks. Thus, only the constant term is affected by uncertainty.

[21] Models with large numbers of state variables can be solved without much computational effort.

$$V_t \equiv E_t \left[\sum_{i=1}^{2} \eta_i \sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j}) \right]$$

where η_i are the weights on households' utilities. Where $U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{i,j}^{1+\varphi_L}}{1+\varphi_L}$.

We measure welfare as the conditional expectation at time zero ($t = 0$), time in which all state variables of the economy equal their steady state values. Since different policy regimes, even not affecting the non-stochastic steady state, are associated with different stochastic steady states, in order to not neglect the welfare effects during the transition from one to another steady state, we use a conditional welfare criterion. Thus, we evaluate welfare conditional on the initial state being the non stochastic steady state[22].

We evaluate the optimal setting of monetary policy in the constrained class of simple interest rate rules.

$$R_t = \Theta(X)$$

Where X represent easily observable macroeconomic indicators tested as possible arguments of the rule:

$$X = \left[R_{t-1}, \frac{\pi_t}{\pi_{ss}}, \frac{y_t}{y_{ss}}, \frac{q_t}{q_{ss}}, \frac{b_{2t}}{b_{2ss}} \right]$$

As implementability condition is required policies to deliver local uniqueness of the rational expectations equilibrium. Following SU we require that the associate equilibrium be locally unique. The configuration of parameters satisfying the requirements and yielding the highest welfare gives the optimal implementable rule. In characterizing optimal policy we search over a grid considering different ranges of the parameters. Then, we compute the total welfare associated to the different parametrizations of the rule.

[22] An alternative to condition on a particular initial state could be to condition on a distribution of values for the initial state. Anyway, when there is a time-inconsistency problem, the optimality of the rule may depens on the initial conditions. A way to overcome this problem could be to find the rule that would prevail under commitment from a "timeless perspective" see Giannoni and Woodford (2002).

6 Optimal Simple Rules

In order to investigate how monetary policy should be optimaly designed in a model with credit frictions, we maximize the households' total welfare with respect to the coefficients of same simple monetary policy rules. As in the monetary business cycle literature, we assume that the nominal interest rate responds to inflation, output and lagged interest rate. In the first case we also assume that the monetary authority has a target on housing prices. Thus, we search over the coefficient of this simple interest rate rule - α_π, α_y, α_R and α_q - using a grid from -4 to 4 for α_π and α_y, from 0 to 0.9 for α_R and from 0 to 1 for α_q[23]. Tab2 summarize the main findings. The optimal rule is characterized by a strong response to inflation deviations from its target. In fact, α_π equals the upper limit of its parameter space. Interest rate smoothing is also optimal. On the contrary, it's not optimal to react to output. These results are consistent with the one obtained by Schmitt-Grohe and Uribe (2004). They show that it is optimal to responde to deviations of output from potential output but not to output variations. While the concept of "output gap" is well understood in models characterized only by inefficiencies related to price stickiness, the definition of potential output in our economy is still not clear.

Optimization over this simple rule shows that the central bank should not take into account variations of housing prices from the steady state level. This means that housing prices are not the right variable to reduce the distortion generated by credit frictions. This results is in line with Faia and Monacelli (2004). Looking at optimal simple rules in a world characterized by credit frictions at the firms' level, they find that an automatic reaction of the interest rate to asset prices variations is not welfare improving.

Now, we investigate if the household indebtedness level is more directly related to the presence of financial frictions in the economy. Thus we also maximize over α_b using a grid from -2 to2. As a results, monetary policy reactions to deviation of the level of indebtedness from the steady state level, turns out to be optimal. As already shown in other studies, an higher indebtedness level induce an increased sensitivity of the household sector to changes in the state of the economy[24]. Greater levels of indebtness may reduce the ability of households

[23] We consider 25 linearly spaced points for each coefficient.

[24] See among others, Debelle (2004) and Mendicino (2004).

to smooth temporary negative shocks due to the burden of the debt. When housing prices fall as a consequence of negative shocks hitting the economy (i.e. negative income or technology shock) the effects of this shocks is also transmitted by the presence of credit constraints. In fact a reduction in asset prices implies a more limited availability of credit to the private sector. If borrowers are highly leveraged, not only they have a more limited access to the credit market, but they also have to face an higher burden of the last period debt.

$$\begin{bmatrix} \textbf{Housing Prices} \\ \hat{R}_t = \alpha_R \hat{R}_{t-1} + \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \alpha_q \hat{q}_t \\ \alpha_R = 0.9 \qquad \alpha_\pi = 4 \qquad \alpha_y = 0 \qquad \alpha_q = 0 \\ TOTAL\ WELFARE = \text{-1} \end{bmatrix}$$

$Tab.2$

$$\begin{bmatrix} \textbf{Indebtedness} \\ \hat{R}_t = \alpha_R \hat{R}_{t-1} + \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \alpha_q \hat{b}_t \\ \alpha_R = 0.9 \qquad \alpha_\pi = 4 \qquad \alpha_y = 2 \qquad \alpha_q = -1.2 \\ TOTAL\ WELFARE = \text{-0.96} \end{bmatrix}$$

$Tab.3$

7 Conclusions

We study optimal monetary policy rules in an economy with credit market frictions at the household level and heterogeneous agents. The aim of the paper is to asses the role of household indebtedness and housing prices in designing monetary policy. As a result, an optimally designed simple monetary policy rule should not take into account deviations of the housing prices from the steady state level. On the other hand, the presence of credit frictions in the economy gives a role to the households' indebtedness level. In fact, interest rate's reactions to variations in the private sector's indebtedness level turns out to be welfare improving. The optimality of an automatic reaction of the interest rate to variation in the level of indebtedness, is given to the fact that higher levels of debt obtained in periods of stable economic conditions are then harmful when market conditions reverse. In fact, the burden of the debt limits the ability of households to smooth negative shocks. Similar results are obtained by Faia and Monacelli (2004) that study welfare based optimal monetary policies in a model with asset prices and credit frictions for firms.

References

[1] Aoki, Kosuke, James Proudman and Jan Vlieghe (2001), "Houses as Collateral: Has the Link between House Prices and Consumption Changed?", mimeo, Bank of England.

[2] Bernanke, Ben S., and Mark Gertler (1989), "Agency Costs, Net Worth and Business Fluctuations," American Economic Review, 79, March, 14-31.

[3] Bernanke, B., M. Gertler and S. Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework", in J.B. Taylor, and M. Woodford, eds., Handbook of Macroeconomics, Amsterdam: North-Holland.

[4] Chari, V.V. and P.J. Kehoe (1998), "Optimal Fiscal and Monetary Policy", Federal Reserve Bank of Minneapolis Research Department Staff Report 251.

[5] Debelle Guy (2004), "Macroeconomic Implications of rising household debt", BIS wp 153.

[6] Faia E. and T. Monacelli (2004), "Welfare-Maximizing Interest Rate Rules, Asset Prices and Credit Frictions", mimeo.

[7] Gilchrist, Simon, and John V. Leahy (2002), "Monetary Policy and Asset Prices", Journal of Monetary Economics, 49, 75-97.

[8] Khan, A., R. King and A.L. Wolman (2000), "Optimal Monetary Policy", Federal Reserve of Richmond w.p. No. 00-10.

[9] Kiyotaki, N. and J. Moore (1997), "Credit Cycles", Journal of Political Economy, 105, April 211-48.25

[10] Iacoviello Matteo(2003) , "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle" Boston College wp.

[11] Jappelli, Tullio, and Marco Pagano (1989), "Aggregate consumption and capital market imperfections: an international comparison", American Economic Review, December, 79, 5, 1088-1105.

[12] Kask J., Household Debt and Finacial Stability, Kroon&Economy n4, 2003.

[13] Poterba, James, (2000), "Stock Market Wealth and Consumption". Journal of Economic Perspectives, 14, Spring, 99-118.

[14] Schmitt-Grohe, S. and M. Uribe (2001), "Optimal Fiscal and Monetary Policy Under Sticky Prices", Journal of Economic Theory.

[15] Schmitt-Grohe, S. and M. Uribe (2001), "Optimal Fiscal and Monetary Policy Under Imperfect Competition", mimeo, Rutgers University and University of Pennsylvania.

[16] Schmitt-Grohe, S. and M. Uribe (2003), "Optimal Simple Monetary and Fiscal Rules in An Economy with Capital", mimeo, Duke University.

WORKING PAPER
PUBBLICATI

1997

1. M. D'Antonio, M. Scarlato
   Struttura economica e commercio estero: un'analisi per le province italiane
2. P. Garegnani, A. Palumbo
   Accumulation of capital
3. E. Cerrito
   Crisi di cambio e problemi di politica monetaria nell'Italia di fine Ottocento.
   Appunti su alcune evidenze empiriche

1998

4. F. Manni
   Struttura delle fonti di finanziamento:
   un'indagine sulle principali società italiane produttrici di bevande
5. J.O. Berger, J. Mortera
   Default Bayes factors for one-sided hypothesis testing
6. A. Trezzini
   Capacity utilisation in the log run: a reply to Serrano
7. A.M. Ferragina
   Quality product differentiation in CEE-EUIntra-Industry trade
8. M.F. Renzi, L. Cappelli, G. Salerno
   Outsourcing: opportunità e limti per le aziende che operano
   con sistemi di qualità conformi alle norme ISO 9000
9. M. Scarlato
   The impact of international trade on employment and wage differentials:
   some evidence from the italian macro-regions
10. A. Trezzini
    Some notes on long-run capacity utilisation,
    steady state and induced investment

1999

11. S. Monni
    A convergence analysis of human development
12. G. M. Rey
    Informazione e politiche pubbliche: non è mai troppo tardi

2000

13. M.M. Barbieri, C. Conigliani
Fractional bayes factors for the analysis of autoregressive models with possible unit roots
14. M. Scarlato
Capitale sociale e Sviluppo Economico
15. A.M. Ferragina
Price versus quality competition in Italy's trade with Central and Eastern Europe over Transition
16. M. D'Antonio, M. Scarlato
Capitale umano e sviluppo economico. Un modello di equilibrio economico generale per il Centro–Nord ed il Mezzogiorno d'Italia
17. M. Cenci, L. Foffo Ciucci
Gli effetti della regolamentazione sull'attività di Insider Trading
18. A. Gheno
Alberi binomiali e struttura della volatilità
19. M. Cenci
Modelli per la struttura a termine con volatilità stocastica (una rassegna critica)
20. M. Cenci, A. Gheno
Metodologie per la valutazione delle obbligazioni convertibili in ipotesi di evoluzione stocastica della struttura per scadenza

2001

21. M. D'Antonio
L'interdipendenza tra economia ed ambiente: un'analisi applicata all'Italia
22. A. Naccarato
Uno studio sulla diffusione di fenomeni metereologici mediante particolari modelli spazio–temporali
23. M. Cenci, M. Scarlato
Settore sommerso e politiche di emersione: un approccio stocastico
24. S. Fadda
Per un nuovo sostegno allo sviluppo locale: patti territoriali e servizi al territorio nel nuovo scenario economico

2002

25. F. De Filippis, D. Storti
Le politiche di sviluppo rurale nell'Unione Europea: un "secondo pilastro" tutto da inventare
26. F. De Filippis, L. Salvatici
The eastwards Enlargement of the European Union and the Common Agricultural Policy
27. S. Fiumara, E. Bellisario
La rilevanza della certificazione di qualità nella responsabilità del produttore di giocattoli

28. S. Fadda
Does the change of Economic institutions require a change in values?
29. A. Corese
La mediateca: approcci gestionali
30. S.M. Fratini
Il teorema dell'indice e l'unicità dell'equilibrio con produzione: una discussione dei limiti di procedimento
31. C. Conigliani, F. Spezzaferri
An alternative Bayes factor for testing for unit autoregressive roots
32. M. Ciurlia
Applicazione di tecniche di simulazione alla valutazione delle opzioni russe

2003

33. C. D'Apice
La distribuzione del reddito in Italia nell' ultimo trentennio
34. C. Conigliani, A. Tancredi
Semi–parametric modelling for costs of healt care technologies
35. M. Cenci, M. Scarlato
Innovazione tecnologica e offerta di skills: una simulazione del ruolo della storia e delle aspettative in un'area in via di sviluppo

2004

36. S. Fratini
Il ritorno delle tecniche in un modello di equilibrio intertemporale con generazioni sovrapposte
37. V. Guizzi
Selezione di portafogli ottimi con vincolo di shortfall
38. A. Lamberti, A. Naccarato
Il problema delle stime preliminari nelle indagini congiunturali: una breve rassegna ed una proposta
39. M. Corradini
Estensione della tecnica degli alberi bi/tri–nomiali ad alberi N–nomiali. Applicazione ai processi diffusivi con salto
40. M. Corradini
Un modello per la dinamica del prezzo a pronti dell'elettricità
41. V. Costantini, S. Monni
Measuring Human and Sustainable Development: An Integrated Approach for European Countries
42. C. Mendicino, A. Pescatori
Credit Frictions, Housing Prices and Optimal Monetary Policy Rules
43. C. Mendicino
Credit Market Development, Economic Performance and Business Cycle Volatility
44. A. Bonfiglioli, C. Mendicino
Financial Liberalization, Bank Crises and Growth: Assessing the links

Finito di stampare nel mese di dicembre del 2004
dalla tipografia « Grafica Editrice Romana S.r.l. » di Roma
per conto della « Aracne editrice S.r.l. » di Roma