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Dipartimento di Economia
Università degli Studi Roma Tre
Via Silvio D'Amico, 77 - 00145 Roma
Tel. 0039-06-57335655 fax 0039-06-57335771
E-mail: dip_eco@uniroma3.it

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UNDERGROUND LABOR, SEARCH FRICTIONS AND MACROECONOMIC FLUCTUATIONS

Giuseppe Ciccarone - Francesco Giuli - Enrico Marchetti

Underground labor, search frictions and macroeconomic fluctuations[1]

Giuseppe Ciccarone
Department of Economics and Law, Sapienza University of Rome.

Francesco Giuli
Department of Economics, "Roma Tre" University

Enrico Marchetti
Department of Economic Studies "S. Vinci", University of Naples Partehnope.

Abstract

We study the effects of underground activities on labour market dynamics in a RBC model with search frictions in the labor market, bargained wage and quadratic hiring costs. Underground activities, which allow agents to (partially) evade taxes, are modelled through a moonlighting production scheme where both regular and underground labor use the same capital equipment inside the firm. Calibrating the model on the U.S. economy, we show that a higher relative size of underground production implies lower average employment and a lower job finding rate, together with higher volatility of employment and lower volatilities of hours worked and wages of regular labor services. The theoretical explanation we provide is that a higher level of the underground activity increases the ratio of the flow contribution of non-working to the flow contribution of a worker to a labour match.

JEL Classification: E320, E260, J640.

Keywords: underground activities, tax evasion, search and matching, real business cycle.

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1 Introduction

The underground economy is a multifaceted object of analysis. Firstly, in spite of recent attempts to provide a clear-cut distinction between formal and informal sectors (Sarte 2000; Koreshkova 2006; Turnovsky and Basher 2009), the existing studies employ different definitions of the underground economy, which can be roughly separated between: (a) the unrecorded economy, which has to do with undeclared production (and so undeclared work and tax evasion) and trade of legal goods and services;[2] (b) the illegal economy, which refers to illegal economic activities. Secondly, the etherogeneous definitional domains of the subject matter imply different methods employable to measure the underground economy, whose size remains largely controversial, especially in the more peripheral countries. Thirdly, different attempts to model the underground economy lead to conflicting evidence on the existing relation between the underground economy and labour market variables.

The aim of this paper is to evaluate the effects of underground activities on labour market volatilities in a dynamic general equilibrium model of the real business cycle type,[3] enriched with search frictions in the labor market, bargaining wage determination in the regular sector and quadratic hiring costs. We assume the existence of three types of agents − firms, households and a government − one homogeneous consumption good and three production factors: regular labor, underground labor and capital. The Government levies taxes on regularly produced income flows and on regular labor services, and balances its budget (in expected terms) in each period. Firms and households, being subject to distortionary taxation, use underground labor input to evade taxes. The Government fights tax evasion through a policy strategy based on penalties and controls influencing the probability of detecting tax evading firms.

The attempts to include the underground sector in a dynamic framework have usually characterized the labor market as split into two separate sectors, one for regular and one for underground labor (see, e.g., Castillo and Montoro 2010; Lisi 2010; Boeri and Garibaldi 2005). Alternatively, search and matching models have been designed so as to consider the hiring of a generic type of labor input which can be endowed, after some shocks, with regular or underground labor "contracts" (Bosch and Esteban-Pretel 2009). These modelling choices are however unable (at least in developed economies) to account for the well documented ability of a relevant share of firms to operate simultaneously in official and unofficial sectors (see, e.g., World Bank 2000). This motivates our preference to model only one productive sector where a *moonlighting production scheme* allows firms to use the same capital stock in both regular and underground production activities, and to evade taxation on the production they hide from official recording.[4] This production scheme, that focuses on undeclared work rather than illegal production, allows us to study the dynamic response of the model's variables to a neutral technology shock, a case which cannot be analyzed by two-sector models. We also analyze their reaction to an idiosyncratic shock hitting only the regular sector and compare our results with those of existing contributions (e.g., Busato and Chiarini 2004).

By calibrating the model on the U.S. economy, we show that the size of the underground activities influences the main labour market variables. More in particular, when moonlighting activities and search frictions are considered, a higher relative size of underground production implies lower average employment and a lower job finding rate, together with higher volatility of employment and lower volatilities of hours worked and wages of regular labor services. The theoretical explanation we provide is that a higher level of the underground activity increases the ratio of the flow contribution

[2] For instance, Renooy et al. (2004) define undeclared work as "productive activities that are lawful as regards their nature but are not declared to the public authorities, taking into account the differences in the regulatory systems between [European Union] Member States".

[3] In line with the scheme adopted in Busato et al. (2011); see also Chiarini et al. (2009).

[4] This concept hence differs form that of ghost production activities, which refers to firms operating exclusively in the underground economy and which is better suited to describe the informal sector in developing countries, where it includes wide areas of home production. Busato et al. (2011) argue that the moonlighting production scheme can have a technological advantage over ghost production due to the presence an aggregate capital externality.

of non-working to the flow contribution of a worker to a labour match, which the recent debate on the "Shimer puzzle"[5] has shown to be one of the main determinants of labor market volatilities.

The paper is organized as follows. In section 2 we describe the model economy and determine wages and hours for both regular and underground labor services. Section 3 presents the main steady state relationships of the model. In section 4 we illustrate our parameterization strategy, compare the relative volatilities produced by the benchmark model with their empirical counterparts and discuss the dynamic response of the model's variables to various types of shocks. In section 5 we carry out a sensitivity analysis able to show the way in which the size of the underground sector affects the labour market variables and provide a theoretical explanation for these results. Section 6 concludes.

2 The model economy

2.1 Matching process and employment dynamics

We model search and matching in the labor market in a standard fashion (e.g., Shimer 2005a). Firms choose the number of vacancies they want to post, the hiring of workers is costly and the existence of search frictions prevents some workers from finding jobs and some posted job vacancies from being filled.

Denoting with U_t and V_t the unemployed workers in search of jobs and the number of posted vacancies, respectively, the number of new matches is determined by the Cobb-Douglas matching function $M_t = M(V_t, U_t) = \eta(V_t)^{\xi}(U_t)^{1-\xi}$, where η is a scale parameter and ξ is the elasticity of the matching function with respect to V_t. It follows that $q_t = M_t/V_t$ is the probability that a firm matches with a worker and $p_t = M_t/U_t$ is the probability that a worker matches with a firm. Furthermore, by defining as $\theta_t = V_t/U_t$ the aggregate labor market tightness (from the viewpoint of the firm), it follows that $p_t = \theta_t q_t$. If the labor market tightness increases, the probability that a firm (worker) finds a worker (firm) diminishes (increases).

The initial period unemployment is given by:

$$U_t = 1 - N_t$$

As we assume that the hiring costs paid by firms are non linear, it is useful to express the hiring rate as the ratio between new matches and the existing employment level:

$$x_t = \frac{q_t V_t}{N_t}$$

Given that existing matches might be severed for exogenous reason at the end of any given period and that new matches become productive in the next period, employment evolves according to the following dynamic equation:

$$N_t = (1-\delta)N_{t-1} + x_{t-1}N_{t-1} \tag{1}$$

where δ denotes the exogenous probability of separation.

[5] See Shimer (2005a). The puzzle is related to the difficulty of both Mortensen-Pissarides and standard RBC models with search frictions to replicate the high volatilities of the unemployment rate and vacancies observed in the data (see: Merz 1994; Andolfatto 1996). The models developed to tackle this issue have followed different routes, including structural elements, such as staggered wage bargaining (Gertler et al. 2008), and specific calibration strategy (Hagedorn and Manovskii 2008a).

2.2 Households

The household sector is composed of a continuum of homogeneous household members who can be either employed or searching for a job. As usual in the literature, we assume that household members pool their resources in order to provide a mutual perfect insurance against differences in their labor income. This hypothesis assures that each household solves the same consumption problem. The representative agent has the utility function:

$$\mathcal{U}\left(C_t, h_{U,t}, h_{U,t}\right) = \log(C_t) - \mathcal{V}_t \tag{2}$$

$$\mathcal{V}_t = B_0 \frac{\left(h_{M,t} + h_{U,t}\right)^{1+\psi_0}}{1+\psi_0} + B_1 \frac{\left(h_{U,t}\right)^{1+\psi_1}}{1+\psi_1}, \quad \psi_1, \psi_0, B_0, B_1 \geq 0,$$

where C_t denotes the household's consumption flow, $h_{M,t}$ and $h_{U,t}$ are the number of hours worked in regular and underground activities, respectively, and $B_0 \frac{(h_{M,t}+h_{U,t})^{1+\psi_0}}{1+\psi_0}$ represents the overall disutility of work. The term $B_1 \frac{(h_{U,t})^{1+\psi_1}}{1+\psi_1}$ reflects the idiosyncratic disutility from hours worked in underground production, which may be associated with the lack of social and health insurance when performing these activities and with subjective costs due to a "social stigma".[6] This formulation allows us to capture the relationship between household utility and the different features of the two types of labor services.[7] As it is difficult to obtain reliable empirical estimates of the supply elasticity of regular and underground labour services, in the subsequent analysis we adopt the specification $\psi_0 = \psi_1 = \psi$ for the disutility of work.

The representative household evades income taxes by reallocating labor services from regular to underground labor activities. We denote with $w_{M,t}$ and $w_{U,t}$ the regular and the underground wages, while r_t is the capital remuneration rate. As the underground-produced income flows $w_{U,t}h_{U,t}$ are not subject to the distortionary income tax rate τ_Y, the budget constraint reads:[8]

$$C_t + K_{t+1} = (1-\tau_Y)\left(w_{M,t}N_t h_{M,t} + (r_t + \delta_K) K_t\right) + (1-N_t)\, b + w_U N_t h_{U,t} + \Pi^f + (1-\delta_K)K_t \tag{3}$$

where K_t is the capital stock, δ_K denotes the capital stock depreciation rate, the price of consumption is normalized to one and $\Pi^f = E_t \pi_t$ is the aggregate amount of expected profits accruing to the household (which the household takes as given). The term b represent the (publicly financed) unemployment insurance, which encompasses all the resources devoted to income support (see the discussion in Hagedorn and Manovskii 2008a).

The representative household maximizes:

$$\mathcal{U}_t\left(K_t\right) = \underset{C_t, K_{t+1}}{Max} \left[\log(C_t) - \mathcal{V}_t + \beta E_t \mathcal{U}_{t+1}\left(K_{t+1}\right)\right] \qquad \text{s.t.: eq (3)}$$

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[6] See, for instance: Benjamini and Maital (1985); Gordon (1989); al-Nowaihi and Pyle (2000); Dhami and al-Nowaihi (2007).

[7] From a technical perspective, it can be easily verified that the chosen utility function is concave, being its Hessian matrix definite negative. For positive quantities : $c, h^i_M, h^i_U > 0$, the Jacobian matrix J and the Hessian one, H, are equal to:

$$J = \begin{bmatrix} c^{-1} \\ -B_0(h^i_{M,t} + h^i_{U,t})^{\psi_0} \\ -B_0(h^i_{M,t} + h^i_{U,t})^{\psi_0} - B_1(h^i_{U,t})^{\psi_1} \end{bmatrix}; \quad H = \begin{bmatrix} -c^{-2} & 0 & 0 \\ 0 & b & b \\ 0 & b & b+d \end{bmatrix}$$

where $b = -\psi_0 B_0 (h^i_{M,t} + h^i_{U,t})^{\psi_0 - 1} < 0$; $d = -\psi_1 B_1 (h^i_{U,t})^{\psi_1 - 1} < 0$. It is hence: $z'Hz = -c^{-2}\left(z_1\right)^2 + b\left(z_2 + z_3\right)^2 + d\left(z_3\right)^2 < 0$ for all non-zero $z \in \mathbb{R}^3$.

[8] Note that the tax rate τ_Y only applies to declared income flows. Further, we here abstract from a value added tax, which could of course be part of an optimal tax policy in the presence of underground activities (although this tax can also be evaded).

where $0 < \beta < 1$ is the subjective discount rate and E_t is the expectation operator.

The optimal household's choice over consumption and capital accumulation leads to the following optimality conditions:[9]

$$\left\{ \begin{array}{c} (C_t)^{-1} = \beta E_t \left(\frac{\partial \mathcal{U}_{t+1}}{\partial K_{t+1}} \right) = \mu_t \\ \beta E_t \left\{ \mu_{t+1} \left[(1-\delta_K) + (1-\tau_Y)(r_{t+1}+\delta_K) \right] \right\} = \mu_t \\ \lim_{t\to\infty} E_0 \mu_t K_t = 0 \end{array} \right\} \tag{4}$$

where μ_t is the shadow price of wealth. The marginal rates of substitutions between consumption and hours worked in the regular and underground sectors are:

$$\begin{aligned} MRS_{M,t} &= -\frac{\partial \mathcal{U}_t / \partial C_t}{\partial \mathcal{U}_t / \partial h_{M,t}} = C_t B_0 \left(h_{M,t} + h_{U,t} \right)^{\psi} \\ MRS_{U,t} &= -\frac{\partial \mathcal{U}_t / \partial C_t}{\partial \mathcal{U}_t / \partial h_{U,t}} = C_t \left[B_0 (h_{M,t} + h_{U,t})^{\psi} + B_1 (h_{U,t})^{\psi} \right] \end{aligned}$$

2.3 Firms

In each period t the representative competitive firm combines hired capital with regular and underground labor services according to the constant returns to scale production function:

$$Y_t = A_t (K_t)^{\alpha} \left[(1-\omega) \left(\vartheta_t \cdot N_t h_{M,t} \right)^{\rho} + \omega \left(N_t h_{U,t} \right)^{\rho} \right]^{\frac{1-\alpha}{\rho}} \tag{5}$$

where $A_{t,}$ is a stochastic variable capturing common technology shocks, and ϑ_t is a stochastic variable that captures idiosyncratic shocks hitting only the regular production activities. The parameters $0 < \alpha < 1$ and $0 < \rho < 1$ are related to the elasticities of substitution between inputs, whereas $0 < \omega < 1$ allows us to differentiate the production shares accruing to the two types of labour. The CES specification of (5) allows us to analyze the substitutability between regular inputs (regular labor and capital) and underground labor services, coherently with our focus on the moonlighting production scheme in which both types of labor use the same capital stock.[10]

As in Busato et al. (2011), firms try to evade taxes on labor by allocating a share of their demand for labor services to underground activities. If a firm is detected evading (with exogenous probability p_D), it must pay the statutory tax rates on labor, τ_N, augmented by the surcharge factor, $s_D > 1$, and obtains profits $\pi_{D,t}$. If a firm is not detected evading (with probability $1-p_D$), it obtains profits $\pi_{ND,t}$. These profits are defined as:

Detected $\sim p_D$	$\pi_{D,t} = Y_t - (1+\tau_N) w_{M,t} N_t h_{M,t} - (1+s_D \tau_{N,t}) w_{U,t} N_t h_{U,t} - (r_t + \delta_K) K_t - hir_t$
Not **D**etected $\sim (1-p_D)$	$\pi_{ND,t} = Y_t - (1+\tau_N) w_{M,t} N_t h_{M,t} - w_{U,t} N_t h_{U,t} - (r_t + \delta_K) K_t - hir_t,$

where hir_t is the firm hiring cost, which we assume to be a quadratic function of the firm's hiring rate, x_t, and of employment, as in Trigari and Gertler (2009):[11]

$$hir_t = \frac{\kappa}{2} (x_t)^2 N_t \tag{6}$$

[9] Detailed calculations are provided in a technical appendix available from the authors upon request.

[10] The production function (5) is reminiscent of that employed by Krusell et al. (2000) and Lindquist (2004) in a different setting with skilled/unskilled labour. It is also similar to that used in Chiarini et al. (2009).

[11] It will become clear in section (4) that we choose this formulation because it allows us to better isolate the effects on labor market volatilities of the parameters affecting the undeground activities in the stationary state.

The firm's expected profits are hence:

$$E_t\pi_t = Y_t - (1+\tau_N)w_{M,t}N_t h_{M,t} - (1+p_D s_D \tau_N)w_{U,t}N_t h_{U,t} - (r_t+\delta_K)K_t - \frac{\kappa}{2}(x_t)^2 N_t \quad (7)$$

and its choice problem is given by:

$$F_t^V(N_t) = \max_{x_t,K_t}\left[E_t\pi_t + \beta E_t\frac{\mu_{t+1}}{\mu_t}F_{t+1}^V(N_{t+1})\right]$$
$$s.t.\ (5),\ (1)$$

where F_t^V is the firm's value. By using the first order condition with respect to the hiring rate $\left(\kappa x_t = E_t\beta\frac{\mu_{t+1}}{\mu_t}\frac{dF_{t+1}^V}{dN_{t+1}}\right)$ and the envelope theorem we get the *job creating condition*:

$$\kappa x_t = \beta E_t\frac{\mu_{t+1}}{\mu_t}\left[\begin{array}{c}\frac{\partial Y_{t+1}}{\partial N_{t+1}} + \frac{\kappa}{2}x_{t+1}^2 - (1+\tau_N)w_{M,t+1}h_{M,t+1} \\ -(1+p_D s_D\tau_N)w_{U,t+1}h_{U,t+1} + (1-\delta)\kappa x_{t+1}\end{array}\right] \quad (8)$$

Equation (8) states that the condition to post an additional vacancy at time t depends on the firm's discounted stream of future earnings and of savings on the hiring process.

The firm rents capital services according to:

$$\frac{\partial Y_t}{\partial K_t} = \alpha\frac{Y_t}{K_t} = r_t + \delta_K$$

2.4 Government budget and aggregate resources constraints

Indicating the aggregate variables with upper case notation, the government budget constraint reads:

$$\tau_Y\left[w_{M,t}N_t h_{M,t} + (r_t+\delta_K)K_t\right] + s_D p_D \tau_N w_{U,t}N_t h_{U,t} + \tau_N N_t h_{M,t}w_{M,t} = b(1-N_t) + G_t \quad (9)$$

where the left-hand side represents the expected government revenues and the right-hand side is composed of unemployment benefits, $b(1-N_t)$, and wasteful expenditure, G_t. The government balances its budget in expected terms since the tax revenues collected from the underground corporate sector also depend on the probability of detection, p_D.

Using the government budget constraint (9) together with the aggregate consumers' budget constraint, obtained from equation (3), and with firms' aggregate profits, obtained from equation (7), we obtain the aggregate resource constraint:

$$C_t + I_t + G_t = Y_t - \frac{\kappa}{2}x_t^2 N_t$$

2.5 Wages and hours determination: underground activities

In line with the existing literature,[12] we assume that regular wages and hours are determined through an efficient Nash bargaining process. We assume instead that underground wages and hours are competitively determined. Hence, when bargaining on $w_{M,t}$ and $h_{M,t}$, the representative agent takes $w_{U,t}$ and $h_{U,t}$ as given. This choice for hours and wages setting is coherent with the moonlighting production scheme adopted in our model economy. While the agreement on regular labor services is subject to a normative set-up which forces agents in an istitutionalized bargaining process, the

[12] See, e.g., Trigari (2009) and Thomas (2008).

absence of the same institutional framework in the underground sector leaves room for competitive forces to act.[13]

As for the determination of underground activities, the firm chooses the desired level of $h_{U,t}$ by maximizing the value of the marginal worker employed in production:

$$\max_{h_{U,t}} \frac{\partial Y_t}{\partial N_t} - (1+\tau_N)w_{M,t}h_{M,t} - (1+p_D s_D \tau_N)w_{U,t}h_{U,t} + \frac{\kappa}{2}x_t^2 + (1-\delta)\,\kappa x_t$$

where $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$ are taken as given. The optimality condition is:

$$w_{U,t} = \frac{(1-\alpha)}{(1+p_D s_D \tau_N)} MP_{U,t} \tag{10}$$

where:[14]

$$MP_{U,t} = \frac{\partial Y_t}{\partial (N_t h_{U,t})} = \frac{Y_t}{N_t h_{U,t}} \left(\frac{(1-\alpha)\,\omega\,(h_{U,t})^\rho}{(1-\omega)\,(\vartheta_t^\rho)\,h_{M,t}^\rho + \omega\,(h_{U,t})^\rho} \right)$$

The household chooses the desired level of $h_{U,t}$ (for given $w_{U,t}$, $w_{M,t}$ and $h_{M,t}$) by maximizing (2) subject to the constraint (3). This gives the optimality condition:

$$w_{U,t} = MRS_{U,t} \tag{11}$$

By equating the two optimality conditions, we obtain the (privately efficient) allocation rule for underground hours worked:

$$MRS_{U,t} = \frac{(1-\alpha)}{(1+p_D s_D \tau_N)} MP_{U,t} \tag{12}$$

2.6 Wages and Hours determination: regular activities

In order to set regular wages and hours, firms and workers maximize (with respect to $w_{M,t}$ and $h_{M,t}$) the Nash product of the surpluses arising from a match: $(S_t^W)^d (S_t^F)^{1-d}$, where $S_t^F = \frac{\partial F_t^V}{\partial N_t}$ and $S_t^W = \frac{\partial \mathcal{U}_t}{\partial N_t}/\mu_t$ denote the firms' and the households' surplus from an employment relationship, and $(1-d)$ and d are the bargaining powers of firms and households, respectively. The value of a match for a firm implies that:

$$\begin{aligned} S_t^F \quad &= \quad \frac{\partial Y_t}{\partial N_t} - (1+\tau_N)w_{M,t}h_{M,t} - (1+p_D s_D \tau_N)w_{U,t}h_{U,t} \\ &\quad -\frac{\kappa}{2}x_t^2 + \beta\,(1-\delta+x_t)\,E_t \frac{\mu_{t+1}}{\mu_t} S_{t+1}^F \end{aligned} \tag{13}$$

The total value of a match for the worker (i.e., the difference between the value of employment and the value of unemployment) is obtained from the household's maximization problem.[15]:

$$\begin{aligned} S_t^W \quad &= \quad (1-\tau_Y)\,w_{M,t}h_{M,t} + w_U h_{U,t} - \frac{B_0(h_{M,t}+h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - \frac{B_1(h_{U,t})^{1+\psi}}{(1+\psi)\,\mu_t} - b + \\ &\quad \beta(1-\delta-\theta_t q_t)E_t \frac{\mu_{t+1}}{\mu_t} S_{t+1}^W \end{aligned} \tag{14}$$

[13] Other papers on underground economy and search frictions, such as Castillo and Montoro (2010) and Boeri and Garibaldi (2005), assume that both underground and regular wages are determined through a Nash bargaining process. Whereas this approach is coherent with their assumption that there exist two distinct labor markets, it would be unfit in our moonlighting scheme.

[14] See the table in Appendix 1.

[15] See appendix 2.

The firm and the worker bargain over both the regular input and the wage so as to maximize their weighted joint surplus:

$$\underset{w_{M,t},h_{M,t}}{Max}\ (S_t^W)^d (S_t^F)^{1-d}$$

taking $w_{U,t}$ and $h_{U,t}$ as given. From the first order condition with respect to regular hours, we obtain the (privately efficient) allocation rule for regular hours worked:

$$MP_{M,t} = \frac{(1+\tau_N)}{(1-\tau_Y)(1-\alpha)} MRS_{M,t} \tag{15}$$

which can be used to derive the equilibrium condition:[16]

$$h_{R,t}^{\rho-1} = \vartheta_t^{\rho} \left(\frac{1-\omega}{\omega}\right) \frac{(1-\tau_Y)(1+p_D s_D \tau_N)}{(1+\tau_N)} \left[1 + \frac{B_1}{B_0}\left(1 + h_{R,t}^{-1}\right)^{-\psi}\right] \tag{16}$$

where $h_{R,t} = h_{U,t}/h_{M,t}$.

This equation shows that in our model the ratio of underground and regular hours emerging from the bargaining and the market solutions is always positive and unaffected, along the cycle, by the common technology shock A_t. The value of $h_{R,t}$ depends instead on the exogenous values of the technology and preference parameters, and on the idiosyncratic shock on the productivity of regular hours, ϑ_t^{ρ}.[17]

From the optimality condition of the Nash bargaining problem with respect to w_M, we obtain the bargained regular wage rate:[18]

$$\begin{aligned} w_{M,t} &= \frac{(1-d)}{(1-\tau_Y)\,h_{M,t}}\left(\frac{\mathcal{V}_t}{\mu_t} + b - w_U h_{U,t}\right) \qquad (17)\\ &+\frac{d}{(1+\tau_N)h_{M,t}}\left(\frac{\partial Y_t}{\partial N_t} + \frac{\kappa}{2}x_t^2 - (1+p_D s_D \tau_N) w_{U,t} h_{U,t} + CV_t\right) \\ \text{where} \quad : \quad & CV_t = \kappa x_t \theta_t q_t \end{aligned}$$

Equation (17) shows that the bargained regular wage is a weigthed average of the reservation values of households and firms. The former value is given by the sum of the overall disutility of work plus the foregone flow benefit from unemployment net of what the household earns by working underground. The firm's reservation value is given by the sum of the marginal revenue and the variation in the hiring cost net of the labor cost in the underground sector, plus the expected net present value of remaining in the match (CV_t). Further, the regular wage is affected by the bargaining power, whereas optimal regular hours are independent from d. Equation (17) implies that, other things being equal, the regular wage is negatively related to the underground one. This is due to the fact that part of the overall disutility of work and marginal product of labor is generated by underground activities and there exists substitutability between regular and underground labor inputs. This implies that, when agents try to strike a balance between the net benefits of working in the two sectors, the prices of the two types of labor will be inversely related. The effect is not symmetric because distortionary taxation and law enforcement introduce a wedge between the compensation and the cost of working underground.

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[16] See appendix 3.

[17] It clearly also depends on the deterrence parameters (p_D, s_D) and on the tax rates τ_Y and τ_N. Long or short term changes in these exogenous variables hence translate into changes in the proportion $h_{R,t}$.

[18] See appendix 3.

3 Stationary state

It should be noted that, as it typically occurs in models with the hiring cost function (6), in our economy the stationary value of the hiring rate x^* (starred variables denote stationary values) is univocally pinned down by the value of δ:

$$0 = -N^* + (1-\delta)N^* + x^* N^* \quad \rightarrow \quad x^* = \delta$$

Further, as in Trigari and Gertler (2009), three of the main labor market variables, θ^*, V^* and q^*, are residually determined once the stationary values of the other variables are obtained. As shown in Appendix 5, where the complete derivation of the steady state is presented, we can in fact formulate the following restricted system of equations:

$$\begin{aligned} J\left(h_M^*, w_M^*\right) &= \left(\frac{1}{\beta} - (1-\delta)\right)\kappa\delta - \frac{\kappa}{2}\delta^2 \\ w_M^* &= w\left(p^*, h_M^*\right) \\ c\left(N^*, h_M^*\right) &= 1 - g\left(w_{M;}^*, h_M^*, p^*\right) - \frac{\kappa}{2}\frac{\delta^2}{\upsilon h_M^*} - \frac{\delta_K \alpha}{r^* + \delta_K} \\ \frac{\delta}{p^*} &= \frac{1-N^*}{N^*} = \frac{1}{N^*} - 1 \end{aligned}$$

where $J\left(..\right)$, $w\left(...\right)$, $c\left(..\right)$ and $g\left(..\right)$ are continuos functions of h_M^*, p^*, w_M^* and N^*. Having determined through this system the stationary values of these four variables, the stationary matches M^* are fixed by:

$$0 = -N^* + (1-\delta)N^* + M^* \quad \rightarrow \quad M^* = \delta N^*$$

and the stationary vacancies are derived from the matching function:

$$V^* = \left(\frac{M^*}{\eta(1-N^*)^{1-\xi}}\right)^{1/\xi}$$

As a consequence, the parameter η cannot affect the labor market variables, but those which are directly related to the stationary vacancies V^*, i.e., θ^* and q^*.

4 Baseline parameterization and simulation

4.1 Parametrization

In order to evaluate the model's performance, we calibrate the model on the US economy. We target the stationary unemployment rate to 11%, as in den Haan et al. (2000) and similarly to models that include the flows in and out of the labour force into the mass of searchers,[19] or that take into account a labor-leisure choice (Walsh 2005, p. 838). We choose a target of 0.71 for the long-run vacancy filling rate q^*, as in den Haan et al. (2000). From steady state computations, the values of the stationary job-finding rate and the market tightness are $p^* = 0.5$ and $\theta^* = 0.7$. The first one is close to the 0.45 value commonly used in the literature (see, e.g. Shimer 2005a, 2005b) and the second one is slightly higher than the value ($\theta = 0.634$) found by den Haan et al. (2000). We target the overall time devoted to work, $h_U^* + h_M^*$, at 0.66, which can be interpreted as 2/3 of the overall available day-time.[20] Given our focus on the moonlighting production scheme, the ratio of

[19] See, e.g., Andolfatto (1996); den Haan et al. (2000) and Trigari (2009).

[20] From steady state computations we also obtain $h_U^*/h_M^* = 0.3$. We accept this number in the abscence of of unambiguous evidence for the average amount of undergound working hours.

underground income over total output, S_U^*, plays a particularly important role in our analysis. We define:

$$S_U^* = \frac{w_U^* h_U^* N^*}{Y^*}$$

and target its value on the basis of the most recent estimates provided by Schneider et al. (2010), who compute for the USA, in the period 1996-2007, a ratio of the shadow economy to total output ranging from 8.4% to 8.8%. We take the average value $S_U^* = 8.6\%$. The following table summarizes our calibration targets:

Table 1: target variables

$U^* = 0.11$	$q^* = 0.71$	$h_U^* + h_M^* = 0.66$	$S_U^* = 0.086$
$p^* = 0.5$	$\theta^* = 0.7$	$h_U^*/h_M^* = 0.3$	

We fix most of the model's parameters in accordance with various sources of independent empirical evidence and with the existing literature. Table 2 synthesizes our benchmark parameterization:

Table 2: benchmark parametrization

Technology	$\alpha = 0.33$	$\omega = 0.3197$	$\rho = 0.56$	$\delta_K = 0.025$
	$\vartheta = 1$	$\kappa = 5$		
Preferences	$B_0 = 1.77$	$B_1 = 0.35$	$\psi = 1.5$	$\beta = 0.99$
Tax structure	$\tau_Y = 0.1186$	$\tau_N = 0.153$	$p_D = 0.057$	$s_D = 1.75$
Labor market	$\xi = 0.5$	$\delta = 0.0625$	$\eta = 0.597$	$d = 0.2$
	$b = 0.3647$			

The calibration of the parameters that are more closely related to tax evasion and the underground economy are drawn form Busato et al. (2011) and deserve particular attention. For the probability of being detected p_D, we rely on Joulfaian and Rider (1998), who estimate the probability of auditing in the United States between 4.6% and 5.7%. We choose the higher value, $p_D = 0.057$, but results do not significantly change if we consider lower values. The Internal Revenue Service has recently undertaken a nationally coordinated strategy aimed at addressing tax evasion schemes (see, e.g., Internal Revenue Service Public Announcement Notice 97-24). Violations of the Internal Revenue Code may result in civil penalties, which include a fraud penalty of up to 75% of the underpayment of tax attributable to the fraud, in addition to the taxes owed. Therefore we set the surcharge factor $s_D = 1.75$. As for the average-long run levels of taxation, the effective income tax rate τ_Y is taken from the Effective Tax Rates, 1979-1997, Table H-1a, prepared by the Congressional Budget Office, and set equal to $\tau_Y = 0.1186$. The statutory tax rate on labour is calculated from the Social Security Administration data from the years 1990s onwards,[21] which imply $\tau_N = 0.153$.

As for preferences and technology, the disutility parameters in the utility function B_0 and B_1, the elasticity parameter ψ and two of the technology parameters, ω and ρ, are set so as to match the target value of $S_U^* = 8.6\%$. The value of the elasticity parameter $\psi = 1.5$ is inside the broad range of empirically plausible values $\psi \in [0; 10]$. We choose a discount rate $\beta = 0.99$, a value commonly used in calibration exercises for the U.S. economy. Finally, the parameters $\alpha = 0.33$ and $\delta_K = 0.025$ are set in accordance with the existing literature, and the relative productivity parameter ϑ^* is set at 1.

[21] Available at: http://www.ssa.gov/oact/ProgData/taxRates.html

As for the labor market parameters, we set $\xi = 0.5$, coherently with the empirical evidence documented in Petrongolo and Pissarides (2001), and $\eta = 0.597$ so as to match, in steady state computations, the targeted values of $q^* = 0.71$ and $p^* = 0.5$. The separation rate is fixed according to the value used in Costain and Reiter (2008), who calibrate an annual job loss rate of 25%, so as to obtain $\delta = 0.0625$ at quarterly frequency (see also Silva and Toledo 2009; Fujita and Ramey 2011). The choice of the remaining parameters, d, κ and b, deserves a more thorough discussion, as they are crucial for the volatilities of the employment rate and vacancies. As highlighted by several studies,[22] the volatilities of the percent deviation of N, V, U and θ from the steady state strongly depend on the value of the unemployment insurance b that, in its turn, affects the "replacement rate" which is relevant for the agents' choices. In a basic model with the extensive margin only (i.e., without explicit utility costs related to the supply of working hours) this quantity is simply the stationary ratio b/w^*. However, as argued by Hagedorn and Manovskii (2008a),[23] the parameter b should also include a number of other elements, as a richer model would also "*incorporate curvature in aggregate productivity and in the utility derived from consumption and leisure, heterogeneity of preferences and workers' productivity, home production, spousal labor supply, etc.*" [Hagedorn and Manovskii 2008a, p.1696] As our model takes into account both the intensive and the extensive margin, together with a fraction of the non officially-accounted income (the underground economy), we hence consider for the calibration of d, κ and b, the ratio of the steady state flow value of non-working to the steady state flow value of the contribution of a worker to the match, corrected by the adjustment for the two tax rates (as in Hagedorn and Manovskii 2008b):

$$f_Q^* \equiv \left(\frac{1+\tau_N}{1-\tau_Y}\right) \frac{\mathcal{V}^*/\mu^* + b - w_U^* h_U^*}{\frac{\partial Y}{\partial N}^* - (1 + p_D s_D \tau_N) w_U^* h_U^* + \frac{\kappa}{2} x^{*2}} \equiv \frac{Q_N^*}{Q_D^*} \left(\frac{1+\tau_N}{1-\tau_Y}\right) \tag{18}$$

The calibration of this replacement rate - which we name "flows ratio" - is a highly controversial and debated issue. Hagedorn and Manovskii (2008a), for example, set this value at 0.955 and emphasize that the closer is the value of f_Q^* to unity,[24] the more is the bargaining framework able to produce realistic volatilities of the labor market variables. Other authors have however proposed different calibration schemes. Shimer (2005a) uses the value 0.4 in a model in which there is no explicit intensive margin and the numerator of f_Q^* includes only b, interpreted as unemployment benefit; on the other hand, Hall (2008) proposes $f_Q^* = 0.7$ in a model in which utility from leisure is explicitly included, and Gertler et al. (2008) provide an estimate of 0.723 in a model without the intensive margin. We choose b and κ so as to obtain $f_Q^* = 0.89$, which is in the range of values [0.723, 0.955] mentioned above. This choice also imply a stationary value of the hiring cost on output, $\frac{\kappa}{2} x^{*2} N^*/Y^*$, equal to 0.92%, which is close to the value of 1% chosen by Walsh (2005). Finally, we set $d = 0.2$ which is smaller than the conventional value 0.5, but higher than the value adopted by Hagedorn and Manovskii's (2008a).

4.2 Simulations and moment analysis

Given our aims, the benchmark parameterization provides a satisfactory simulation of the main relative volatilities in comparison with their empirical counterparts. Actual data for the United States economy are seasonally adjusted series from 1964:1 to 2010:2 expressed in constant 2005 prices.[25] As for the empirical series of the (log of) unemployment level, Yashiv (2005) has pointed out that, when considering the pool of unemployed persons to be compared with the same variable

[22] See, e.g., Yashiv (2005); Mortensen and Nagypal (2007); Costain and Reiter (2008); Hagedorn and Manovskii (2008a); Pissarides (2009); Gertler and Trigari (2009).

[23] Togheter with other authors, such as Mortensen and Nagypal (2007) and Hall (2008).

[24] Together with a very small value of d.

[25] Except for the data of the job findng probability p, which cover the period from 1964:1 to 2004:4. See the appendix for a description of the data sources and of the transformation adopted in order to compare the empirical series with

in a theoretical model, pools of workers outside the labor force should be taken into account.[26] For this reason we adopt a procedure similar to that of Yashiv (2005) (p. 918, table 1, series 2) and construct the series for the total amount of unemployment, U_{10}, by adding to the official pool of unemployment, U_0, a fraction (10%) of the working age population, POP_{+16}, which is neither employed, N , nor unemployed:

$$U_{10} = U_0 + 0.1\left(POP_{+16} - N - U_0\right) \tag{19}$$

We hence assume that in each period a share of workers from the out of the labor force pool adds to that of of searches. The resulting series U_{10} implies an average unemployment rate of 11%, equal to the target value of Table 1.[27]

Table 3 summarizes the results of the benchmark simulation (hatted variables indicate percentage deviations from the steady state and σ_X indicates the percentage standard deviation of variable X).[28]

Table 3: simulated vs. actual series

Variable	Model (A_t) ρ_A=0.9; σ_A=0.0036	Model (ϑ_t) ρ_ϑ=0.9; σ_ϑ=0.0065	U.S. Data
$\sigma_{\widehat{Y}}$	1.57	1.57	1.57
$\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$	3.94	3.84	3.92
$\sigma_{\widehat{N}}/\sigma_{\widehat{Y}}$	0.49	0.47	0.66
$\sigma_{\widehat{w}_M+\widehat{h}_M}/\sigma_{\widehat{Y}}$	0.43	0.58	0.59
$\sigma_{\widehat{p}}/\sigma_{\widehat{Y}}$	4.48	4.37	6.35
$\sigma_{\widehat{w}_M}/\sigma_{\widehat{Y}}$	0.35	0.29	0.52
$\sigma_{\widehat{h}_M}/\sigma_{\widehat{Y}}$	0.20	0.43	0.27
$\sigma_{\widehat{V}}/\sigma_{\widehat{Y}}$	5.26	5.13	8.61
$\sigma_{\widehat{\theta}}/\sigma_{\widehat{Y}}$	9.00	8.77	12.28
$\sigma_{\widehat{h}_R}/\sigma_{\widehat{Y}}$	—	1.12	—
$\sigma_{\widehat{S}_U}/\sigma_{\widehat{Y}}$	—	0.93	—
$\sigma_{\widehat{C}}/\sigma_{\widehat{Y}}$	0.42	0.41	0.80
$\sigma_{\widehat{I}}/\sigma_{\widehat{Y}}$	2.51	2.48	4.70

The second column of Table 3 shows the simulation results when the model is fed with a neutral technology shock ($\varepsilon_{A,t}$) on A_t, a case which cannot be analyzed by two-sector models, whereas the third column shows the results obtained when only an idiosyncratic shock ($\varepsilon_{\vartheta,t}$) on ϑ_t is active, as in Busato and Chiarini (2004). The stochastic components are modelled as AR(1) processes:

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \varepsilon_{A,t}; \qquad \widehat{\vartheta}_t = \rho_\vartheta \widehat{\vartheta}_{t-1} + \varepsilon_{\vartheta,t};$$

where the shocks $\varepsilon_{A,\vartheta}$ are white noise with zero mean and constant variances $\sigma_{A,\vartheta}$. In both cases, the model can account for a significant fraction of the volatilities of $\widehat{N}_t$, $\widehat{V}_t$ and $\widehat{\theta}_t$. As for the volatility

the simulated ones. Values of original data are first expressed in logs and then HP filtered with a smoothing parameter equal to 1600.

[26] This is due to the fact that: i) there are sizeable flows between the pool out of the labor force and the labor force, including flows directly to and from employment; ii) the unemployment status in current surveys may be misclassifyed, so that many of those who are not in the labor force are in a situation equivalent to the unemployed; iii) the out of the labor force flows exhibit markedly different cyclical properties relative to flows between employment and unemployment. See Yashiv (2005), pp. 917-8.

[27] This is also coherent with the fact that our model includes a labor-leisure choice. Also notice also that in RBC models with a labor-leisure chioce it is usual to distinguish between the pool of searchers and the official amount of unemployed.

[28] The figures in the columns "Model" refer to the average relative standard deviations calculated on $5,000$ draws of time-series of the same length as in the data, keeping parameters fixed at the calibrated values of Table 2.

of unemployment, the model's simulation very closely matches the empirical value of $\sigma_{\widehat{U}_{10}}/\sigma_{\widehat{Y}}$. If the official unemployment were considered, the empirical relative standard deviation would be: $\sigma_{\widehat{U}_0}/\sigma_{\widehat{Y}} = 7.37$.

The simulated values also replicate the high contemporaneous correlation between $\widehat{U}_t$ and $\widehat{V}_t$: the correlation coefficient emerging from the empirical data is equal to -0.906 and the simulated value is -0.903 in case of a shock on A_t and to -0.875 under a shock on ϑ_t. The high relative variation of the job finding rate $\frac{\sigma_p}{\sigma_Y}$ is also coherent with the empirical estimation.

Even though the neutral technology shock cannot produce, by construction, any volatility in the ratio h_R (see equation (16)) and in the share of the overall underground wage income to total output, $S_{U,t} = \frac{w_{U,t}h_{U,t}N_t}{Y_t}$, cyclical movements in these two quantities can be easily generated by the consideration of the idiosyncratic shock affecting the relative productivity parameter ϑ_t. In this way, as shown by equation (16), the quantities describing the relative weight of the underground sector over the regular one also fluctuate over the cycle. In this case, we obtain that both $\widehat{h}_R$ and $\widehat{S}_U$ show a significant volatility, which is substantially equal to that of the GDP, together with a correlation coefficient between $\widehat{h}_U$ and $\widehat{h}_M$ equal to -0.87. A consensus on the cyclical properties of the underground sector is lacking in the literature. For example, Bajada (2003) and Giles (1997) argued in favor of a positive comovement between the regular and the underground sector, whereas a number of other authors found an opposite relation.[29] The lack of sufficiently reliable and long time series for the underground sector[30] places the empirical identification of the relevance of different types of shock in driving the business cycle beyond the scope of present paper.

4.3 Impulse response analysis

The dynamic responses of the main model's variables to shocks on both A_t and ϑ_t are shown in figure 1. The response of the variables to the two types of shock are qualitatively the same, but for the reaction of $\hat{h}_U$. The response of most of the relevant variables, such as the aggregate demand components, the labor market probabilities, the labor market quantities and rates, are in line with the existing literature. The economic mechanism behind these IRFs is standard: an increase in labor productivity translates into a positive effect on labor demand, so that production, capital usage and aggregate demand components increase. This is reflected in the behavior of the labor market variables. Worked hours (in particular following a shock on A_t) peak on impact, whereas the adjustment along the extensive margin must undergo the search process and the costly vacancy posting, so that vacancies and employment follow a gradual and persistent transitional dynamics. In a similar way, the job finding rate increases and the vacancy filling rate decreases.

The dynamics of wages and hours in the regular and the underground sectors show instead peculiarities deserving special attention. Following an idiosyncratic shock on ϑ_t, regular hours increase whereas underground hours fall, due to the substitution effect related to the CES production structure. At the same time, the shock on ϑ_t produces the counterintuitive effect of increasing both the regular and the underground wage. The regular wage goes up due to a higher productivity and a tighter labor market. The increase in $\widehat{w}_{U,t}$ is instead due to the composition of two opposite effects which can be singled out by rewriting equation (10) as:

$$w_{U,t} = \frac{(1-\alpha)^2}{(1+p_D s_D \tau_N)} \frac{Y_t}{N_t h_{U,t}} \frac{\omega}{(1-\omega)\left(\frac{\vartheta_t}{h_{R,t}}\right)^{\rho} + \omega}$$

[29]See, e.g., Tanzi (1983); Giles (1999) and Russo (2008), for different countries. See also the discussion in Busato and Chiarini (2004).

[30]Which also reflects the variety of methods proposed for estimating them, e.g., currency demand, electricity use, etc.; see Schneider and Enste (2000).

An increase in ϑ_t reduces the ratio $h_{R,t}$, thus decreasing the term $\frac{\omega}{(1-\omega)(\vartheta_t/h_{R,t})^\rho+\omega}$, but raises the term $\frac{Y_t}{N_t h_{U,t}}$ by decreasing h_U and increasing the overall productivity of employment, as it can be seen by considering the aggregate production function:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha} H_t^{\frac{1-\alpha}{\rho}} \quad \text{where: } H_t = (1-\omega)(\vartheta_t h_{M,t})^\rho + \omega (h_{U,t})^\rho \tag{20}$$

and noting that the reaction of employment productivity, $\partial Y_t/\partial N_t = (1-\alpha)\frac{Y_t}{N_t}$, to a change in ϑ_t is always positive: $\frac{\partial^2 Y}{\partial N \partial \vartheta} > 0$.The composition of the two effects depends on the calibration of parameters' values. In the benchmark model the second effect is the one that prevails.

The effects produced by a positive shock on A_t are different, as in this case the regular and the underground hours increase exactly in the same way, due to the constancy of h_R. The higher common productivity increases both wages. However, whereas $\widehat{w}_{U,t}$ peaks on impact, as it is determined by marginal labor productivity in a perfectly competitive market, the reaction of the regular wage $\widehat{w}_{M,t}$ is hump-shaped and displays substantial persistence. The high degree of rigidity displayed by the behavior of $\widehat{w}_{M,t}$ is testified by a value of its relative variance which is below the empirical one (see table 3). This is due to both the wage bargaining mechanism and the presence of underground activities. The former cause is well known. The latter one, which is instead specific to our model, can be understood by rewriting equation (17) as:

$$\begin{aligned} w_{M,t} \quad = \quad & \frac{(1-d)}{(1-\tau_Y)}\frac{1}{h_{M,t}}\left(\frac{\mathcal{V}_t}{\mu_t}+b\right) + \frac{d}{(1+\tau_N)}\frac{1}{h_{M,t}}\left(\frac{\partial Y_t}{\partial N_t}+\frac{\kappa}{2}x_t^2 + CV_t\right) \\ & - \left[\frac{(1-d)(1+\tau_N) + d(1-\tau_Y)(1+p_D s_D \tau_N)}{(1-\tau_Y)(1+\tau_N)}\right] w_{U,t} h_{R,t} \end{aligned}$$

As $h_{R,t}$ does not react to the common productivity shock, the response of the regular wage is partially dumpened by the increase of the underground wage, the more so the higher is the (positive) value of the coefficient multiplying the term $w_{U,t}h_{R,t}$. We can hence conclude that the underground sector is an additional channel contributing to the rigidity of the regular wage.

5 The impact of underground activities on labor market volatilities

In order to further investigate the impact of underground activities on labor market dynamics, and to confirm that as the share of the underground sector S_U^* is reduced the relative volatility $\sigma_{\widehat{w}_M}/\sigma_{\widehat{Y}}$ increases, in this section we carry out a sensitivity experiment and provide a theoretical explanation for the results we obtain.

We start from the steady state underground share S_U^* , which reads:

$$S_U^* = \frac{(1-\alpha)^2}{(1+p_D s_D \tau_N)}\left(\frac{\omega}{(1-\omega)(h_R)^{-\rho}+\omega}\right)$$

As the stationary level of h_R^* depends only on a limited set of parameters (see equation (16)), it can be shown[31] that a reduction in ω brings about a fall in the stationary underground share S_U^*:

$$\frac{dS_U^*}{d\omega} = (S_U^*)^2 \frac{(1+p_D s_D \tau_N)}{(1-\alpha)^2}\left(\frac{\vartheta}{h_R^*}\right)^\rho \left[\frac{1}{\omega^2} + \frac{\rho}{h_R^*}\left(\frac{1-\omega}{\omega}\right)\frac{dh_R^*}{d\omega}\right] > 0 \quad \text{as it is } \frac{dh_R^*}{d\omega} > 0$$

[31] See footnote 9.

In the CES representation of the production technology (5), ω is directly related to the share of the underground income/product on overall output. We can then interpret a lower value of ω as the representation of an economy requiring less underground labor for structural reasons,[32] e.g., due to a more technologically advanced production structure or to a better rule of law.[33] We may hence deepen our understanding of the role played by the underground sector in shaping the model's dynamics by comparing the relative standard deviations of the main benchmark model's variables with those obtained by setting lower values of ω (Table 4).[34]

Table 4: sensitivity to underground activities' level

	Model:			
	benchmark	$\omega = 0.29$	$\omega = 0.27$	$\omega = 0.26$
variable:				
N^*	89%	92%	93%	94%
p^*	0.5	0.71	0.89	0.99
f_Q^*	0.89	0.87	0.86	0.85
S_U^*	8.6%	6.6%	5.5%	4.9%
$\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$	3.94	4.15	4.09	4.02
$\sigma_{\widehat{N}}/\sigma_{\widehat{Y}}$	0.49	0.36	0.28	0.25
$\sigma_{\widehat{w}_M+\widehat{h}_M}/\sigma_{\widehat{Y}}$	0.43	0.55	0.63	0.66
$\sigma_{\widehat{p}}/\sigma_{\widehat{Y}}$	4.48	4.52	4.37	4.26
$\sigma_{\widehat{w}_M}/\sigma_{\widehat{Y}}$	0.35	0.45	0.51	0.54
$\sigma_{\widehat{h}_M}/\sigma_{\widehat{Y}}$	0.20	0.21	0.22	0.22
$\sigma_{\widehat{V}}/\sigma_{\widehat{Y}}$	5.26	5.07	4.79	4.63
$\sigma_{\widehat{\theta}}/\sigma_{\widehat{Y}}$	9.00	9.06	8.77	8.55

Table 4 shows that as the stationary underground share decreases (to roughly one half of the benchmark value):

- the stationary value of employment N^* increases, bringing about a rise in the job finding rate due to the reduction in the number of searchers, and the flows ratio f_Q^* decreases;
- the volatility of employment $\sigma_{\widehat{N}}/\sigma_{\widehat{Y}}$ monotonically decreases, while those of $\widehat{h}_M$ and $\widehat{w}_M$ increase;
- the volatilities of unemployment, market tightness and vacancies, $\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$, $\sigma_{\widehat{\theta}}/\sigma_{\widehat{Y}}$ and $\sigma_{\widehat{V}}/\sigma_{\widehat{Y}}$, are hump-shaped.[35]

We can hence summarize our main finding as follows: *when moonlighting activities are considered, an economy with a lower degree of underground labor is characterized by lower instability in employment and by higher volatility of regular wages and hours. Furthermore, for an admissible range of stationary values of other labor market variables, the volatilities of unemployment, vacancies and market tightness first increase and subsequently decrease as the underground share raises.* Figure 2 (in the appendix) contrasts the behavior of $\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$ and $\sigma_{\widehat{N}}/\sigma_{\widehat{Y}}$ as functions of ω.

[32] It is a well known empirical finding that more developed economies (and more technologically advanced production sectors) generally have lower shares of underground activities. See, e.g., the summary in Scheider et al. (2010b).

[33] A reduction in S_U^* could of course also be obtained *via* a change in other - deterrence or preference/technological - parameters, such as the tax and penalty rates, or the inverse Frisch elasticities.

[34] Table 4 reports the result obtained with a simulation analogous to the one of Table 3: the model is fed with a shock on A_t and the parameters of the stochastic process are $\rho_A = 0.9$ and $\sigma_A = 0.0036$.

[35] This emerges from a sensitivity analysis more refined than that shown in table 4.

The general explanation of these results is that the reduction in the volatilities of employment and other labor market quantities is driven by a reduction in the ratio of the flow contribution of non-working to the flow value of the contribution of a worker to the match, i.e., f_Q^*. The fall in this ratio is caused, in its turn, by the reduction in ω. The consequent fall in S_U^* is accompanied with a relevant effect on stationary employment/unemployment: U^* drops and N^* raises significantly. Tables 3-4 show that even a relatively tiny underground sector may have an influence on the volatilities of the main labor market variables (both quantities and prices). The explanation of this result can be divided into two items.

1. The presence of underground activities has an unfavorable effect on the total productivity of employment: the higher is the underground economy (the higher is ω), the lower is $\partial Y^*/\partial N^*$, which implies a lower level of stationary employment N^*.
2. A high level of ω also affects the way in which the *total surplus of a match*, to be shared by workers and firms, is divided between its two main components: the net marginal contribution of employment to a match and the net value of non-work activities. When ω is higher, these two quantities change in a way that increases the flows ratio f_Q^* in equation (18) and the volatility of employment.

In order to show point 1, consider equation (20) and note that the reaction of employment productivity $\partial Y^*/\partial N^*$ to a change in ω depends on the ratio $\frac{h_U^*}{h_M^*}$ according to the following inequalities:[36]

$$\frac{\partial^2 Y^*}{\partial N^* \partial \omega} \lesseqgtr 0 \text{ iff } \ h_U^* \lesseqgtr h_M^*$$

Hence, as long as the average regular working time is greater than the underground one (as it occurs in our model), a reduction in ω fosters the stationary productivity of employment. This is due to the fact that when ω decreases, the term H^* in the production function (20) increases because more weight is placed on the more abundant factor (h_M^*), that is: $\frac{\partial H^*}{\partial \omega} < 0$ when $h_U^* < h_M^*$.[37]

In order to explain point 2, we focus on S^T, the total surplus of a match which is shared between workers and firms *via* the Nash bargaining on w_M and h_M; The dynamics of S^T is ruled by these two equations:[38]

$$S_t^T = Q_{D,t} - \left(\frac{1+\tau_N}{1-\tau_Y}\right) Q_{N,t} + \beta\left(1-\delta - dp_t\right) E_t \frac{\mu_{t+1}}{\mu_t} S_{t+1}^T \tag{21}$$

and :

$$\kappa x_t = \beta(1-d) E_t \frac{\mu_{t+1}}{\mu_t} S_{t+1}^T \tag{22}$$

where we also used the definitions in equation (18):

$$\begin{aligned} Q_{N,t} &= \mathcal{V}_t/\mu_t + b - w_{U,t} h_{U,t} \\ Q_{D,t} &= \partial Y_t/\partial N_t - (1 + p_D s_D \tau_{N,t}) w_{U,t} h_{U,t} + \frac{\kappa}{2} x_t^2 \end{aligned}$$

According to (21), the total surplus is composed of: (i) the expected surplus of the match in the next period; (ii) the difference between the marginal contribution of employment to a match, $Q_{D,t}$, and the

[36] See footnote 9.

[37] It is worthnoting that the term H^* acts on the production function in the same way as an externalty term would do in a standard Cobb-Douglas production function of the form $Y = H\left(K^\alpha N^{1-\alpha}\right)$.

[38] See appendix 4.

net value of non-work activities, $(1+\tau_N)\,Q_{N,t}/\,(1-\tau_Y)$, both considered as net of the contributions related to the underground activities (the terms including $w_{U,t}h_{U,t}$) because underground hours and wages are set in a separated (competitive) framework.

Equations (21)-(22) provide the stationary value of the total surplus, S^{T*}:

$$S^{T*} = \frac{\kappa x^*}{\beta(1-d)} \quad \text{which implies:} \tag{23}$$

$$\frac{\kappa x^*}{\beta(1-d)} = \frac{Q_D^* - \frac{(1+\tau_N)}{(1-\tau_Y)}Q_N^*}{1-\beta\,(1-\delta-dp^*)} \tag{24}$$

S^{T*} is hence determined only by a restricted set of parameters being independent, in particular, from those affecting the share of underground activities S_U^* (the parameters included in equation (16)). This property, which is common to the models characterized by the hiring cost function (6), is related to the independence of the stationary value of the hiring rate from those of the other endogenous variables: $x^* = \delta$.[39]

By comparing equations (24) and (18) we get that, in order to obtain a positive total surplus, the flows ratio f_Q^* must be smaller than one:

$$S^{T*} > 0 \rightarrow Q_D^* > \frac{(1+\tau_N)}{(1-\tau_Y)}Q_N^* \rightarrow f_Q^* < 1$$

Furthermore, the reaction of S^{T*} to changes in ω obeys the following rule:

$$S^{T*}d\beta\frac{\partial p^*}{\partial\omega} = \frac{\partial Q_D^*}{\partial\omega} - \frac{(1+\tau_N)}{(1-\tau_Y)}\frac{\partial Q_N^*}{\partial\omega}$$

The reaction of p^* to ω is negative (as confirmed by table 4): $\frac{dp^*}{d\omega} < 0$. This is a direct consequence of what we discussed sub point 1 above: as underground activities expand, productivity and employment fall, and this decreases the job-finding probability p^*. The reactions of the Q_D^* and Q_N^* components to an increase in ω must hence satisfy the following inequality:

$$\frac{\partial Q_D^*}{\partial\omega} < \frac{(1+\tau_N)}{(1-\tau_Y)}\frac{\partial Q_N^*}{\partial\omega} \tag{25}$$

The last step requires to compute (from equation (18)):

$$\frac{\partial f_Q^*}{\partial\omega} = \frac{1}{Q_D^*}\left[\frac{(1+\tau_N)}{(1-\tau_Y)}\frac{\partial Q_N^*}{\partial\omega} - f_Q^*\frac{\partial Q_D^*}{\partial\omega}\right]$$

This equation, together with the conditions $f_Q^* < 1$ and (25), implies that:

$$\frac{\partial f_Q^*}{\partial\omega} > 0 \tag{26}$$

This explains the sensitivity exercise summarized in Table 4, showing that a higher share of underground activities increases the flows ratio f_Q^*. An intuition for this result can be grasped by recalling that $\frac{(1+\tau_N)}{(1-\tau_Y)}Q_N^*$ and Q_D^* represent the value of unemployment for a worker and the net contribution to production, respectively. When ω increase, both derivatives in (25) are negative; the fall in the

[39] This effect is absent in simpler models with linear cost of vacancy posting, such as those discussed in Costain and Reiter (2008). Note however that our main results on the effect of the undergound labor on the volatilities of labor market variables are confirmed also under linear vacancy costs.

value of unemployment is hence lower (in absolute terms) than the fall in the net contribution to production. This is due to the fact that, coherently with the observation sub point 1 above, when underground activities widen, the productivity of employment decreases and the contribution of the worker to production falls. At the same time, the opportunity cost of unemployment is lower and unemployment is relatively more attractive: the lower overall income for the worker partially offsets the effect of ω on the numerator of f_Q^*.

As pointed out by the existing literature (see e.g. Hagedorn and Manovskii 2008a; Costain and Reiter 2008; Mortensen and Nagypal 2007), the ratio f_Q^* is crucial in determining the volatilities of the hiring rate and the other labor market quantities. When the overall benefits of non-working and working are almost equal (the ratio f_Q^* is close to one), firms are almost indifferent with respect to the choice of employing an additional unit of labor. An exogenous productivity shock - which affects both $Q_{N,t}$ and $Q_{D,t}$ - may hence have a strong impact on the choice of hiring, or non hiring, a new worker, thus modifying the overall employment level of the firm and, as a consequence, the other labor market quantities. For the same reason, when f_Q^* is close to one the values of unemployment and employment to the worker are almost equal. As highlighted by Gertler et al. (2008), a small change in the wage hence generates a sizeable percentage change in the relative gains (to the worker) from employment with respect to unemployment and this explains the dampened response of wages to a productivity shock. We have here shown that this ratio is influenced also by the weight of underground activities according to inequality (26).

Finally, the hump-shaped behavior of $\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$ depicted in Figure 2, given the monotonic behavior of $\sigma_{\widehat{Y}}$, is due to the joint effects of two counteracting forces. The volatilities of employment and unemployment are linked *via* the equation:

$$\sigma_{\widehat{U}} = \left(\frac{N^*}{1-N^*}\right)\sigma_{\widehat{N}}$$

so that:

$$\frac{\partial \sigma_{\widehat{U}}}{\partial \omega} = \left(\frac{N^*(\omega)}{1-N^*(\omega)}\right)\left[\frac{\partial \sigma_{\widehat{N}}}{\partial \omega} + \left(\frac{\sigma_{\widehat{N}}(\omega)}{N^*(1-N^*(\omega))}\right)\frac{\partial N^*}{\partial \omega}\right]$$

On the one hand, the derivative of N^* with respect to ω is negative, $\frac{\partial N^*}{\partial \omega} < 0$, because the productivity of employment is increasing in ω (see point 1 above). On the other hand, the derivative $\frac{\partial \sigma_{\widehat{N}}}{\partial \omega}$ is positive; this is explained by the effect of ω on f_Q^*, as discussed sub point 2 above. This result is worth noticing as it shows that a reduction in underground activities may generate a trade-off: the overall stationary employment, together with productivity, improves, but the volatility of unemployment increases. This behavior of $\sigma_{\widehat{U}}$ also explains that of $\sigma_{\widehat{\theta}}$ and $\sigma_{\widehat{V}}$. Intuitively, a higher level of $\sigma_{\widehat{U}}$ adds to the variability of the market tightness and, as a consequence, firms' decisions on vacancy posting, which depend on θ, become more volatile. The opposite occurs when $\sigma_{\widehat{U}}$ decreases.

6 Conclusions

In this paper we have shown that, in a model economy characterized by moonlighting production, tax evasion and search frictions in the labor market, the size of underground production significantly affects both levels and variabilities of the main labour market variables. Numerical simulations show that when the relative size of underground income increases the volatility of employment also increase. This is accompanied by a fall in the volatilities of regular wages and hours worked. Furthermore, whereas an aggregate productivity shock leaves the relative size of the underground activities unchanged, an idiosyncratic productivity shock on the regular labor input makes the same size move countercyclically.

These results can be related to the effect of the underground economy's size on the stationary values of the two main components of the total surplus to be shared by workers and firms in the wage bargaining: the overall benefits of non-working and the worker's contribution to production. An increase in the underground size, by bringing these two quantities closer, makes the firm more sensitive to an exogenous productivity shock and the worker more respondent to a small change in the wage. This mechanism brings about the higher volatilities of employment, and the lower volatility of wages following a productivity shock.

We leave for future investigations the effects of different tax schemes and other types of policies on labour market variables and on the size of underground activities. More in particular, the framework adopted here could contribute to the debate on the best policies to adopt in order to tackle undeclared work, that is, on the relative preference to be attributed to policies aimed at repressing underground work by acting on controls and sanctions, and those targeted at improving labor market performances.

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Appendix

1. CES marginal productivities

From straightforward computation relative to the production function (5), we obtain the following definitions:

$$\begin{aligned}
\frac{\partial Y_t}{\partial N_t} &= (1-\alpha)\frac{Y_t}{N_t} = MP_{N,t} \\
\frac{\partial Y_t}{\partial (N_t h_{M,t})} &= \vartheta_t (1-\alpha)\left(\frac{Y_t}{\vartheta_t N_t h_{M,t}}\right)\frac{(1-\omega)(\vartheta_t N_t h_{M,t})^\rho}{(1-\omega)(\vartheta_t N_t h_{M,t})^\rho + \omega (N_t h_{U,t})^\rho} = MP_{M,t} \\
\frac{\partial Y_t}{\partial (N_t h_{U,t})} &= (1-\alpha)\left(\frac{Y_t}{N_t h_{U,t}}\right)\frac{\omega (N_t h_{U,t})^\rho}{(1-\omega)(\vartheta_t N_t h_{M,t})^\rho + \omega (N_t h_{U,t})^\rho} = MP_{U,t} \\
\left(\frac{\partial Y_t}{\partial N_t}\right)\frac{1}{h_{M,t}} &= (1-\alpha)\frac{Y_t}{N_t h_{M,t}} = MP_{M,t} + \frac{h_{U,t}}{h_{M,t}} MP_{U,t} \\
\left(\frac{\partial Y_t}{\partial N_t}\right)\frac{1}{h_{U,t}} &= (1-\alpha)\frac{Y_t}{N_t h_{U,t}} = MP_{M,t} + \frac{h_{M,t}}{h_{U,t}} MP_{U,t} \\
\frac{\partial^2 Y_t}{\partial N_t \partial h_{X,t}} &= (1-\alpha) MP_{X,t};\ \text{for: } X = M, U
\end{aligned}$$

2. Derivation of the household's suplus

In each period, a fraction of household members, N_t, is employed, earns market and underground hourly wages and incurs in the disutility of work; the remaining fraction $(1-N_t)$ is searching for a job, enjoys leisure time and receives the unemployment benefit b. The change in the household's optimal utility of having an additional member employed is given by the solution of the following maximization problem with respect to the state variable N_t:

$$\begin{aligned}
\mathcal{U}_t (N_t, K_t) &= \underset{N_t}{Max} \left[\log(C_t) - N_t \mathcal{V}_t + \beta E_t \mathcal{U}_{t+1} (N_{t+1}, K_{t+1})\right] \\
\text{s.t.}\quad C_t &= -K_{t+1} + (1-\tau_Y)(w_{M,t} N_t h_{M,t} + (r_t + \delta_K) K_t) + (1-N_t) b + N_t w_U h_{U,t} + \\
&\quad \Pi_i + (1-\delta_K) K_t; \\
N_{t+1} &= (1-\delta) N_t + (1-N_t) p_t
\end{aligned}$$

The first order condition is:

$$\frac{\partial \mathcal{U}_t}{\partial N_t} = (C_t)^{-1}\left[(1-\tau_Y) w_{M,t} h_{M,t} + w_{U,t} h_{U,t} - b\right] - \mathcal{V}_t + \beta(1-\delta-p_t) E_t \frac{\partial \mathcal{U}_{t+1}}{\partial N_{t+1}}$$

By defining $S_t^W = \frac{\partial \mathcal{U}_t}{\partial N_t}/\mu_t$, and recalling that $\mu_t = (C_t)^{-1}$, we obtain equation (14) in the main text.

3. Nash bargaining over regular wages and hours

The first order conditions for $w_{M,t}$ and $h_{M,t}$, from the Nash optimization problem $\underset{w_{M,t},h_{M,t}}{Max} (S_t^W)^d (S_t^F)^{1-d}$ are:

where $w_{U,t}$ and $h_{U,t}$ are taken as given. The first order conditions for $w_{M,t}$ and $h_{M,t}$, are:

$$w_{M,t}: \quad (1-d) j_t^{F,w} \left(S_t^W\right) + d j_t^{W,w} \left(S_t^F\right) = 0 \tag{27}$$

$$h_{M,t}: \quad (1-d) j_t^{F,h}\left(S_t^W\right) + d j_t^{W,h}\left(S_t^F\right) = 0 \tag{28}$$

where

$$j_t^{F,w} = \frac{\partial\left(S_t^F\right)}{\partial w_{M,t}} = -(1+\tau_N) h_{M,t} \tag{29}$$

$$j_t^{W,w} = \frac{\partial\left(S_t^W\right)}{\partial w_{M,t}} = (1-\tau_Y) h_{M,t} \tag{30}$$

$$j_t^{F,h} = \frac{\partial\left(S_t^F\right)}{\partial h_{M,t}} = \frac{\partial^2 Y_t}{\partial N_t \partial h_{M,t}} - (1+\tau_N) w_{M,t} \tag{31}$$

$$j_t^{W,h} = \frac{\partial\left(S_t^W\right)}{\partial h_{M,t}} = (1-\tau_Y) w_{M,t} - MRS_{M,t} \tag{32}$$

are the marginal effects of the bargained variables over the firm's and the household's surpluses.

We now substitute (27) into equation (28) to get:

$$(1-d) j_t^{F,h} \frac{d}{1-d} \frac{(1-\tau_Y)}{(1+\tau_N)} S_t^F + d j_t^{W,h} S_t^F = 0$$

By using (31) and (32), and recalling that $\frac{\partial^2 Y_t}{\partial N_t \partial h_{M,t}} = (1-\alpha) MP_{M,t}$, we obtain equation (15):

$$MP_{M,t} = \frac{(1+\tau_N)}{(1-\tau_Y)(1-\alpha)} MRS_{M,t}$$

Dividing (12) by (15), it turns out that the existing relationship between the regular and underground working hours is:

$$\frac{MP_{U,t}}{MP_{M,t}} = \frac{(1+p_D s_D \tau_N)(1-\tau_Y)}{(1+\tau_N)} \frac{MRS_{U,t}}{MRS_{M,t}}$$

From this equation it is possible to derive the equilibrium relationship (16) for the ratio h_R.[40]

Turning to the determination of the wage in the regular sector, firstly consider that, from equations (29), (30) and (27), the value from employment for the worker at $t+1$ can be written as:

$$S_{t+1}^W = \frac{d}{1-d} \frac{(1-\tau_Y)}{(1+\tau_N)} S_{t+1}^F;$$

Secondly, substituting this equation into (14) together with $\kappa x_t = E_t \beta \frac{\mu_{t+1}}{\mu_t} S_{t+1}^F$ we obtain:

$$\begin{aligned} S_t^W = {} & (1-\tau_Y) w_{M,t} h_{M,t} + w_U h_{U,t} - \frac{B_0 (h_{M,t} + h_{U,t})^{1+\psi}}{(1+\psi)\mu_t} - \frac{B_1 (h_{U,t})^{1+\psi}}{(1+\psi)\mu_t} - b \\ & + \frac{d}{1-d}(1-\delta-\theta_t q_t)\kappa x_t \left(\frac{1-\tau_Y}{1+\tau_N}\right) \end{aligned}$$

Thirdly, using the optimality condition $\kappa x_t = E_t \beta \frac{\mu_{t+1}}{\mu_t} S_{t+1}^F$, equation (13) becomes:

$$S_t^F = \frac{\partial Y_t}{\partial N_t} - (1+\tau_N) w_{M,t} h_{M,t} - (1+p_D s_D \tau_N) w_{U,t} h_{U,t} + (1-\delta)\kappa x_t + \frac{\kappa}{2} x_t^2$$

Finally, substituting the above equations into (27), we get equation (17) in the main text.

[40] See footnote 9.

4. Derivation of the total surplus S^T

Start from the sharing rule (27) of section 2.5, which can be expressed in two equivalent forms:[41]

$$\begin{aligned} S_t^F &= (1-d)\left[S_t^F + \left(\frac{1+\tau_N}{1-\tau_Y}\right)S_t^W\right] \\ S_t^W &= d\left(\frac{1-\tau_Y}{1+\tau_N}\right)\left[S_t^F + \left(\frac{1+\tau_{N,t}}{1-\tau_{Y,t}}\right)S_t^W\right] \end{aligned}$$

These equations allow us to define (as in Hagedorn and Manovskii 2008b) the total surplus of a match as:

$$S_t^T = S_t^F + \left(\frac{1+\tau_N}{1-\tau_Y}\right)S_t^W$$

The firms' and the households' surpluses can hence be written as:

$$S_t^F = (1-d)S_t^T; \qquad S_t^W = d\left(\frac{1-\tau_Y}{1+\tau_N}\right)S_t^T \tag{33}$$

Now, by using equations $\kappa x_t = E_t\beta\frac{\mu_{t+1}}{\mu_t}S_{t+1}^F$, (14), (13) for S^F and S^W and iterating equations (33) one period ahead, we obtain the equations (21) and (22) in the main text.

5. Stationary state

The long-run equilibrium of the labor market is given by:

$$\begin{aligned} 0 &= -N^* + (1-\delta)N^* + xN^*; \quad x^* = \frac{q^*V^*}{N^*}; \quad p^* = M^*/U^*; \\ M^* &= \eta(V^*)^{\xi}(U^*)^{1-\xi}; \quad \theta^* = V^*/u^*; \quad q^* = \frac{M^*}{V^*}. \end{aligned}$$

We aim to show that the labor market variables, θ^*, V^* and q^*, are determined residually once the other main endogenous variables are fixed (as in Trigari and Gertler 2009).

As a first step, notice that the interest rate is determined by the stationary (Euler) equation:

$$r^* = \left[\frac{1}{\beta} - 1 + \delta_K\right]\frac{1}{(1-\tau_Y)} - \delta_K > 0.$$

and that, using the stationary demand for capital, $r^* = \alpha\left(\frac{Y}{K}\right)^* - \delta_K$, it is possible to write:

$$\left(\frac{Y}{K}\right)^* = \frac{r^*+\delta_K}{\alpha} \quad \text{or also:} \quad K^* = \frac{\alpha}{r^*+\delta_K}Y^*$$

Now insert this value of K^* into the production function:[42]

$$Y^* = (K^*)^{\alpha}(N^*)^{1-\alpha}\left[(1-\omega)(h_M^*)^{\rho} + \omega(h_U^*)^{\rho}\right]^{\frac{1-\alpha}{\rho}}$$

so as to obtain:

$$\frac{Y^*}{N^*} = \left(\frac{\alpha}{r^*+\delta_K}\right)^{\frac{\alpha}{1-\alpha}}\left[(1-\omega)(h_M^*)^{\rho} + \omega(h_U^*)^{\rho}\right]^{\frac{1}{\rho}}$$

[41] See Costain and Reiter (2008) and Monacelli et al. (2010), among others.
[42] The stationary values of A and ϑ are equal to 1.

By factoring out h_M^* it is straightforward to show that the ratio $\frac{Y^*}{N^*}$ is proportional to h_M^*:

$$\frac{Y^*}{N^*} = \upsilon h_M^*, \quad \text{with: } \upsilon = \left(\frac{\alpha}{r^* + \delta_K}\right)^{\frac{\alpha}{1-\alpha}} \left[1 - \omega + \omega \left(h_R^*\right)^{\rho}\right]^{\frac{1}{\rho}}$$

as the term h_R^* is univocally set by equation (16) alone. It follows that MP_M^* is determined as:

$$MP_M^* = \frac{Y^*}{h_M^* N^*} \frac{(1-\alpha)(1-\omega)}{1 - \omega + \omega \left(h_R^*\right)^{\rho}} = \upsilon \frac{(1-\alpha)(1-\omega)}{1 - \omega + \omega \left(h_R^*\right)^{\rho}} \tag{34}$$

Then, consider the (stationary) equilibrium condition (10):

$$w_U^* = \frac{1-\alpha}{1 + p_D s_D \tau_N^*} MP_U^* = \frac{Y^*}{\left(1 + p_D s_D \tau_N^*\right) h_U^* N^*} \frac{\omega (1-\alpha)^2}{(1-\omega)\left(h_R^*\right)^{-\rho} + \omega} \tag{35}$$

and substitute it into the job creating condition (8), at the stationary point, together with the ratio $\frac{Y^*}{N^*}$ computed above, so as to obtain:

$$\left[\frac{1}{\beta} - (1-\delta)\right] \kappa x^* - \frac{\kappa}{2} \left(x^*\right)^2 = h_M^* \left[(1-\alpha)\upsilon - (1+\tau_N) w_M^* - \upsilon \frac{\omega (1-\alpha)^2}{(1-\omega)\left(h_R^*\right)^{-\rho} + \omega}\right]$$

As the stationary hiring rate is equal to $x^* = \delta$, the job creating condition can be written as a function $J(\cdot,\cdot)$ of h_M^* and w_M^* only:

$$J\left(h_M^*, w_M^*\right) = \left(\frac{1}{\beta} - (1-\delta)\right) \kappa \delta - \frac{\kappa}{2} \delta^2$$

Now note that the regular wage equation (17) can be written as:[43]

$$\begin{aligned} w_M^* &= d\left(\frac{Q_1}{1+\tau_N}\right) + (1-d)\left(\frac{Q_2}{1-\tau_Y}\right) \\ \text{where } &: \\ Q_1 &= MP_M^* + \alpha MP_U^* h_R^* + \frac{\kappa}{2}\frac{x^{*2}}{h_M^*} + \frac{CV^*}{h_M^*} \\ Q_2 &= \frac{MRS_M^*}{1+\psi} - \psi \frac{MRS_U^*}{1+\psi} h_R^* + \frac{b}{h_M^*} \end{aligned} \tag{36}$$

The marginal rates of substitutions are related to the marginal productivities by the equilibrium equations for working hours:

$$MRS_U^* = \frac{1-\alpha}{1 + p_D s_D \tau_N} MP_U^*; \qquad MRS_M^* = \frac{(1-\alpha)(1-\tau_Y)}{1+\tau_N} MP_M^*$$

By substituting them into (36) we obtain:

$$\begin{aligned} w_M^* = & \frac{d}{1+\tau_N^*}\left(MP_M^* + \alpha MP_U^* h_R^* + \frac{\kappa}{2}\frac{\delta^2}{h_M^*} + \frac{\kappa \delta p^*}{h_M^*}\right) \\ & + \frac{(1-d)}{1-\tau_Y^*}\left(\frac{(1-\alpha)\left(1-\tau_Y^*\right) MP_M^*}{\left(1+\tau_N^*\right)(1+\psi)} - \frac{\psi (1-\alpha) MP_U^* h_R^*}{\left(1+p_D s_D \tau_N^*\right) 1+\psi} + \frac{b}{h_M^*}\right) \end{aligned}$$

[43] We make use of equations (10) and (11), together with the definitions of $MRS_{U,M}$ in the main text. See footnote 9.

As $MP^*_U h^*_R$ is determined:

$$MP^*_U h^*_R = \frac{Y^*}{h^*_M N^*} \frac{\omega (1-\alpha)}{(1-\omega)(h^*_R)^{-\rho} + \omega} = \upsilon \frac{\omega (1-\alpha)}{(1-\omega)(h^*_R)^{-\rho} + \omega}$$

and MP^*_M is set by equation (34), the stationary, regular wage is a function of the two stationary variables (h^*_M, p^*):

$$w^*_M = w(h^*_M, p^*) \tag{37}$$

Now move to the aggregate demand components. As for public expenditure, make use of the government budget constraint divided by the stationary output:

$$\frac{G^*}{Y^*} = (\tau_Y + \tau_N) \frac{w^*_M N^* h^*_M}{Y^*} + \tau_Y (r^* + \delta_K) \frac{K^*}{Y^*} + s_D p_D \tau_N \frac{w^*_U h^*_U N^*}{Y^*} - b\frac{(1-N^*)}{Y^*}$$

From equation (35), it is: $S^*_U = \frac{w^*_U h^*_U N^*}{Y^*} = \left(\frac{1}{1+p_D s_D \tau_N}\right) \frac{\omega(1-\alpha)^2}{(1-\omega)\left(h^*_R\right)^{-\rho}+\omega}$, so that the stationary ratio of underground income to output is determined. Note also that the ratio $\frac{K^*}{Y^*}$ is determined by the interest rate r^* and the demand for capital. The term $\left(\frac{N^* h^*_M}{Y^*}\right) w^*_M$ depends only on h^*_M and p^*, *via* the bargained wage equation (37). Now consider the term $(1-N^*)/Y^*$ and recall that $\frac{Y^*}{N^*} = \upsilon h^*_M$; this implies: $\frac{1-N^*}{Y^*} = \frac{1}{\upsilon h^*_M}\left(\frac{1-N^*}{N^*}\right)$. But from the labor market equations it is:

$$\frac{x^*}{p^*} = \frac{1-N^*}{N^*}$$

Hence, the term $\frac{1-N^*}{Y^*}$ is a function of h^*_M and p^*. These relations, taken together, imply that the ratio $\frac{G^*}{Y^*}$ is a function of w^*_M, h^*_M and p^*:

$$\frac{G^*}{Y^*} = g(w^*_M, h^*_M, p^*)$$

As for consumption, notice that the underground hours equilibrium condition, $MRS^*_U = \frac{1-\alpha}{1+p_D s_D \tau^*_N} MP^*_U$, can be express it in this way:

$$C^* \left[B_0 \left((1+h^*_R)\right)^{\psi} + B_1 \left((h^*_R)^{\psi}\right] (h^*_M)^{\psi} = \frac{1-\alpha}{1+p_D s_D \tau^*_N} MP^*_U$$

As the term MP^*_U is a function of h^*_M alone,[44] it is possible to re-arrange this equation so as to obtain:

$$\frac{C^*}{Y^*} = \left(\frac{1-\alpha}{1+p_D s_D \tau_N}\right) \frac{(1-\alpha)\,\omega / \left[(1-\omega)\,\vartheta^{\rho} (h^*_R)^{-\rho} + \omega\right]}{\left[B_0 \left((1+h^*_R)\right)^{\psi} + B_1 \left((h^*_R)^{\psi}\right] h^*_R (h^*_M)^{1+\psi} N^*} = c(N^*, h^*_M)$$

which says that the ratio $\frac{C^*}{Y^*}$ is a function of (h^*_M, N^*). Consider now the resource constraint divided by output:

$$\frac{C^*}{Y^*} + \delta_K \frac{K^*}{Y^*} = 1 - \frac{G^*}{Y^*} - \frac{\kappa}{2} (x^*)^2 \frac{N^*}{Y^*}$$

[44] That is: $MP^*_U = \frac{Y^*}{h^*_M h^*_R N^*} \frac{(1-\alpha)\omega}{(1-\omega)\vartheta^{\rho}\left(h^*_R\right)^{-\rho}+\omega}$.

By substituting the functions described above it follows that:

$$c\left(N^{*},h_{M}^{*}\right)+\frac{\delta_{K}\alpha}{r^{*}+\delta_{K}}=1-g\left(w_{M}^{*},h_{M}^{*},p^{*}\right)-\frac{\kappa}{2}\frac{\delta^{2}}{\upsilon h_{M}^{*}}$$

which is an equation in the four variables: (w_M^*, h_M^*, p^*, N^*).

Summing up, we can put together the four equations:

$$\begin{aligned}
J\left(h_{M}^{*},w_{M}^{*}\right) &= \left[\frac{1}{\beta}-(1-\delta)\right]\kappa\delta-\frac{\kappa}{2}\delta^{2}\\
w_{M}^{*} &= w\left(p^{*},h_{M}^{*}\right)\\
c\left(N^{*},h_{M}^{*}\right) &= 1-g\left(w_{M}^{*},h_{M}^{*},p^{*}\right)-\frac{\kappa}{2}\frac{\delta^{2}}{\upsilon h_{M}^{*}}-\frac{\delta_{K}\alpha}{r^{*}+\delta_{K}}\\
\frac{\delta}{p^{*}} &= \frac{1-N^{*}}{N^{*}}=\frac{1}{N^{*}}-1
\end{aligned}$$

to obtain the same system of equations presented in section 3.

6. Sources of data and data transformations

GDP, total consumption and investment are taken from the Bureau of Economic Analysis (BEA) database: NIPA tables 1.1.6; the price deflator P_Y is taken from NIPA table 1.1.9. Employment (N) and population aged 16 and older (POP_{+16}) are taken from the Bureau of Labour Statistics, Current Population Surveys (BLS-CPS) . Hours worked (H) are average weekly hours by the non farm civilian employees, also taken from BLS-CPS. The overall (regular) wage income, W, is taken from the "wage accruals" series in BEA-NIPA table 1.1.2; these values are deflated using the price deflator P_Y and divided by the product $N \times H$, so as to obtain the hourly real wage rate $(w_{M,t})$. Official unemployment (U_0) is taken from BLS-CPS - total amount of unemployed persons. The series is then used to compute the "adjusted" unemployment level U_{10}, according to equation (19) discussed in the main text. As a proxy for vacancies (V), we use the Help Wanted Index (HWI) collected by the Conference Board and made available by the OECD (MEI dataset: http://stats.oecd.org/).[45] This value is used to compute the market tightness: $\theta = (HWI/U_{10})$. As for the job finding probability p we rely on Shimer's calculations[46] and take the subset of observations from 1964:1 to 2004:4.

Table A1 below summarizes the data sources and transformations of original time series used to obtain the results contained in Table 3 in the main text. Hatted variables indicate the cyclical component after the H-P filtering with smoothing parameter set at 1600.

[45] The base year for the Help Wanted Index provided by the OECD is 2000.
[46] For details, see Shimer (2005b) and his webpage http://home.uchicago.edu/~shimer/data/flows/.

Table A1 - Sources of data and data transformations

Variable	Source	Definition	Database Table/Code	Transformation
Y	BEA	Real GDP (base year: 2005)	NIPA table 1.1.6	$\widehat{Y}_t = \ln\left(\frac{Y_t}{POP_{+16,t}}\right)$
C	BEA	Real Consumption (base year: 2005)	NIPA table 1.1.6	$\widehat{C}_t = \ln\left(\frac{C_t}{POP_{+16,t}}\right)$
I	BEA	Real Investment (base year: 2005)	NIPA table 1.1.6	$\widehat{I}_t = \ln\left(\frac{I_t}{POP_{+16,t}}\right)$
P_Y	BEA	GDP deflator (base year: 2005)	NIPA table 1.1.9	——
W	BEA	Wage and salary accruals private sector	NIPA table 1.1.2	$\widehat{w}_{M,t} = \ln\left(\frac{W_t/P_{Y,t}}{H_t \times N_t}\right)$
POP_{+16}	BLS	Working age population 16 years and older	LNU00000000	——
H	BLS	Average weekly hours of private sector workers	CES0500000007	$\widehat{h}_{M,t} = \ln(H_t)$
N	BLS	Employed persons total, private non farm sectors	LNS12000000	$\widehat{N}_t = \ln(N_t)$
U_0	BLS	Official unemployment 16 years and over	LNS13000000Q	——
U_{10}	——	Our computations as in equation (19)	——	$\widehat{U}_t = \ln(U_{10,t})$
HWI	OECD	Help Wanted Index from Conference Board	MEI dataset http://stats.oecd.org/	$\widehat{V}_t = \ln(HWI_t)$
θ	——	Market tightness our computation	——	$\widehat{\theta}_t = \ln\left(\frac{HWI_t}{U_{10,t}}\right)$
p	Shimer	Job finding probability R. Shimer computations	R.Shimer webpage http://home.uchicago.edu/~shimer	$\widehat{p}_t = \ln(p_t)$

Figures:

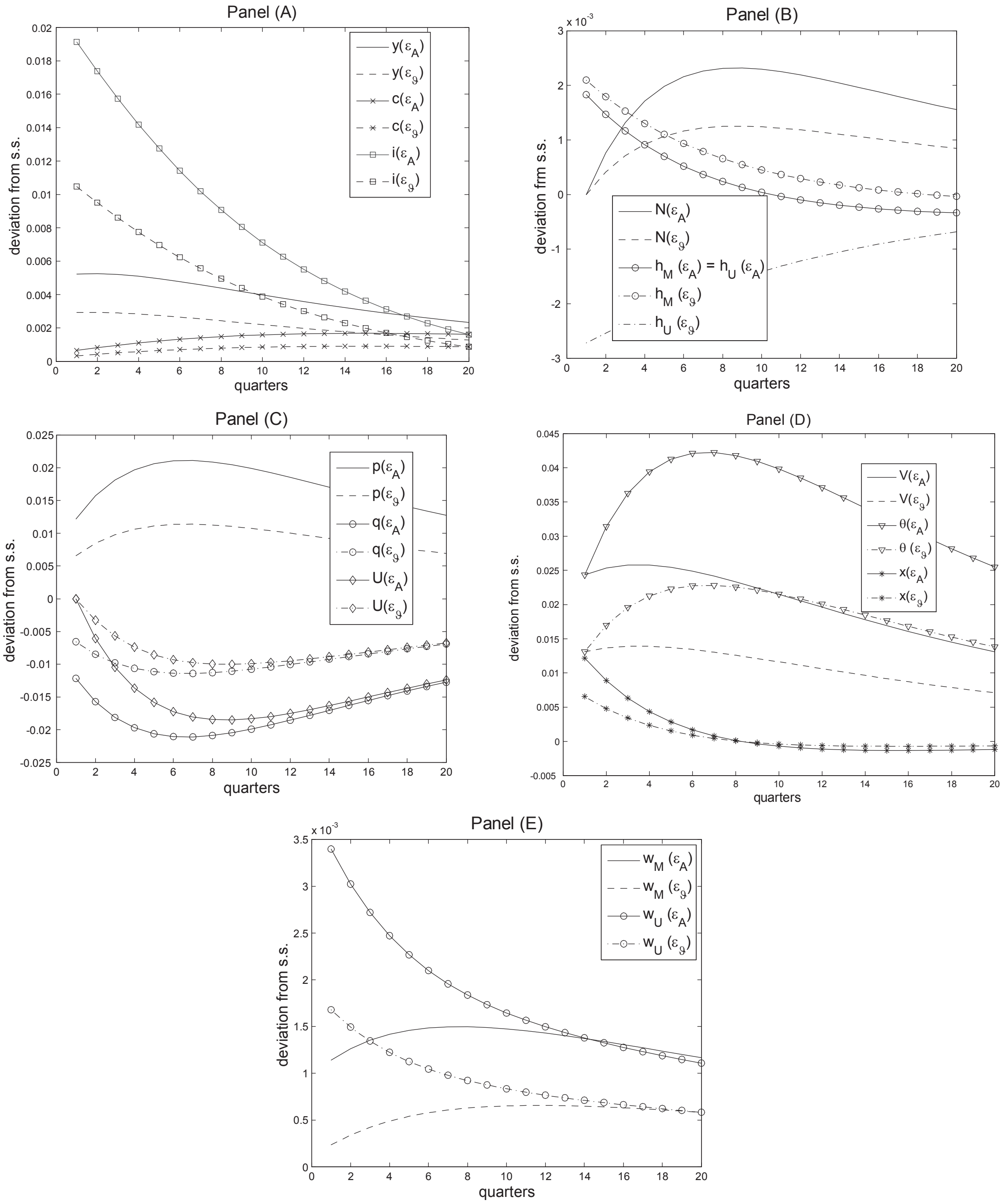

Figure 1: **Impulse response functions:** the figure shows the first 20 quarters response to positive shocks ε_A and ε_ϑ for the benchmark parameterisation of Table 2 (with $\rho_A = \rho_\vartheta = 0.9$; $\sigma_A = \sigma_\vartheta = 0.036$). Variables depicted in panels (A)-(E) correspond to the model's hatted variables.

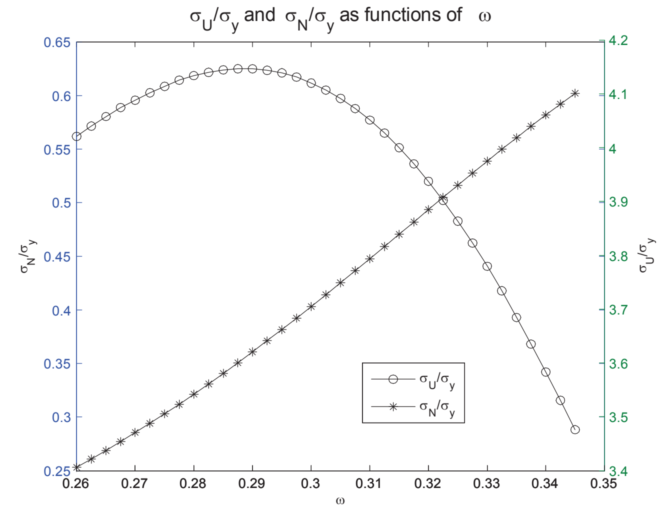

Figure 2: **Unemployment and employment volatilities:** the figure shows the behaviour of $\sigma_{\widehat{U}}/\sigma_{\widehat{Y}}$ and $\sigma_{\widehat{N}}/\sigma_{\widehat{Y}}$ in response to increases in the underground share in the range $\omega \in [0.26;\ 0.345]$. For $\omega < 0.26$ the stationary job finding probability p^* becomes greater than one, whereas for $\omega > 0.345$, the vacancy filling rate q^* becomes greater than one.