UNIVERSITÀ DEGLI STUDI ROMA TRE
DIPARTIMENTO DI ECONOMIA

PORTFOLIO SELECTION: A LINEAR APPROACH WITH DUAL EXPECTED UTILITY

M. Cenci, F. Filippini

Working Paper n° 48, 2005

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Introduction 1
1. Dual expected utility 2
2. The efficient frontier in the dual expected utility 7
3. Some remarks about the parameters 11
4. Application 12
5. Conclusions 16
Appendix A 19
Appendix B 20
Bibliography 21

Abstract. *This paper analyses the portfolio selection problem under the non-expected utility theory. We assume that the decision maker ranks the alternatives by using a specific Dual Expected Utility. This function allows returns which are less than or equal to a fixed benchmark to be weighted in a different way from those greater than the fixed benchmark. In this model the implicit risk measure is more general than the standard deviation and it coincides with the downside risk only due to the appropriate choices of the parameters. Under normally distributed returns and appropriate choices of the benchmark, the approach suggested is equivalent to the Markowitz model in term of efficient frontier and moreover has the advantage of using linear programming to obtain the optimal portfolio. It can thus handle high dimensional problems. We also show results obtained by implementing the model on the Italian stock market.*
(keywords: dual expected utility, portfolio selection, linear programming)

INTRODUCTION

Since Markowitz's pioneering work concerning portfolio selection, many extensions have been proposed to the original model in order to explain the individual asset-holding behaviour and to develop normative rules for asset selection. Markowitz's model is based on the mean-variance portfolio selection, where the average returns and the risk of portfolio are determined in terms of the mean and variance of the stock returns, respectively.

Even if the variance is a correct risk measure in the case of normally distributed returns, unfortunately, in real markets, the stock returns don't follow a normal distribution .

Alternative theories displaying several alternatives risk measures have been developed in literature ([2],[3],[9],[10],[12]). For schemes which can at least partially take account of transaction costs and of constraints associated with minimal transaction lots see ([6],[7],[10]). Many of these extensions propose risk measures which allow the reduction of the original Markowitz quadratic optimisation problem into a linear one.

In this paper we suggest a criterion based on a particular rank-dependent utility function ([14],[19],[20]) for portfolio choice. We assume a risk-averse expected utility maximizer decision maker (DM) and that expected utility is calculated as a weighted mean that overestimates returns under a fixed benchmark and underestimates those over the benchmark.

Due to its simple application to portfolio selection, the particular dual expected utility proposed in this framework appears to be a good compromise between the general rank-dependent expected utility theory and the linear cumulative prospect theory ([6],[15]). In agreement with the above theories, our approach provides a suitable description of individual behaviour because it:

- overcomes the Allais [1] and Ellesberg [4] paradoxes;
- considers the loss aversion hypothesis as in Kahneman and Tversky [16]
- assigns different weights to the outcomes over and under the benchmark [17].

Mathematically, the principal advantage is that, with the proposed dual expected utility function, assuming a known distribution of portfolio returns and an appropriate benchmark, the DM maximises a concave increasing piece-wise linear utility function and the optimisation portfolio problem could be solved using linear programming.

The outline of the paper is as follows: in Section 1 we describe the rank-dependent expected utility and the dual expected utility. In Section 2, we implement the special case of the dual expected utility suggested. Some qualitative properties of the parameters are shown in Section 3. Results of the analysis of stocks included in the S&P MIB Index are shown in Section 4 .Some financial remarks and suggestions for future research are given in Section 5.

1. DUAL EXPECTED UTILITY

The rank dependent expected utility theory (RDEU) is a generalisation of expected utility theory (EU) based on probability weighting. The RDEU may be regarded as an "Expected utility with respect to a transformed probability distribution"[17]

Let X be a random variable whose outcomes x_i occur with probability p_i. Furthermore, we assume $x_1 \le x_2 \le ..... \le x_n$.

For RDEU the decision maker uses an additively separable utility function whose functional form is

$$RDEU(X) = \sum_{i=1}^{n} u(x_i)\gamma(p_1, p_2, ...p_i)$$

where $u(x)$ is the usual utility function,

$$\gamma(p_1, p_2, ...p_i) = g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)$$

and g(.) is a non decreasing function such that g(0)=0 and g(1)=1.

Note that, when $g(x) = x$, $\gamma(p_1, p_2, ...p_i) = p_i$ and $RDEU(X) = EU(X)$.

If $g(x)$ is concave, it overweights the worst outcomes and if $g(x)$ is convex it underweights these outcomes with respect to the best ones.

Moreover, Quiggin ([9]) shows that, if $g(x)$ is monotonic, the choices made according to the RDEU are coherent with stochastic dominance principle.

We propose a generalization of the RDEU in which the ordering of the values of the random variable portfolio returns divides in two separable classes, the returns greater than a fixed benchmark and the outcomes less or equal to a fixed benchmark. We then can express the function $g(x)$ in that, it overweights the worst outcomes and underweights the positive ones.

A suitable $g(x)$ which represents this is as follows

$$g(x)=\begin{cases} Bx & if \quad 0 \le x \le \Pr(X \le V) \\ Ax + C & if \quad \Pr(X \le V) < x \le 1 \end{cases}$$

where V is the benchmark value.

Under concavity and monotonicity of the function, we have:

$$1 \le B \le \frac{1}{\Pr(X \le V)}, \qquad A = \frac{1 - B\Pr(X \le V)}{1 - \Pr(X \le V)}, C = (B - A)\Pr(X \le V).$$

For each outcome of the random variable less or equal to the benchmark, this implies $\gamma(p_1, p_2, ... p_i) = Bp_i$, while for outcomes over the benchmark we have $\gamma(p_1, p_2, ... p_i) = Ap_i$

The function $g(x)$ is given by a piece-wise linear graph, the dashed line of Fig.1.

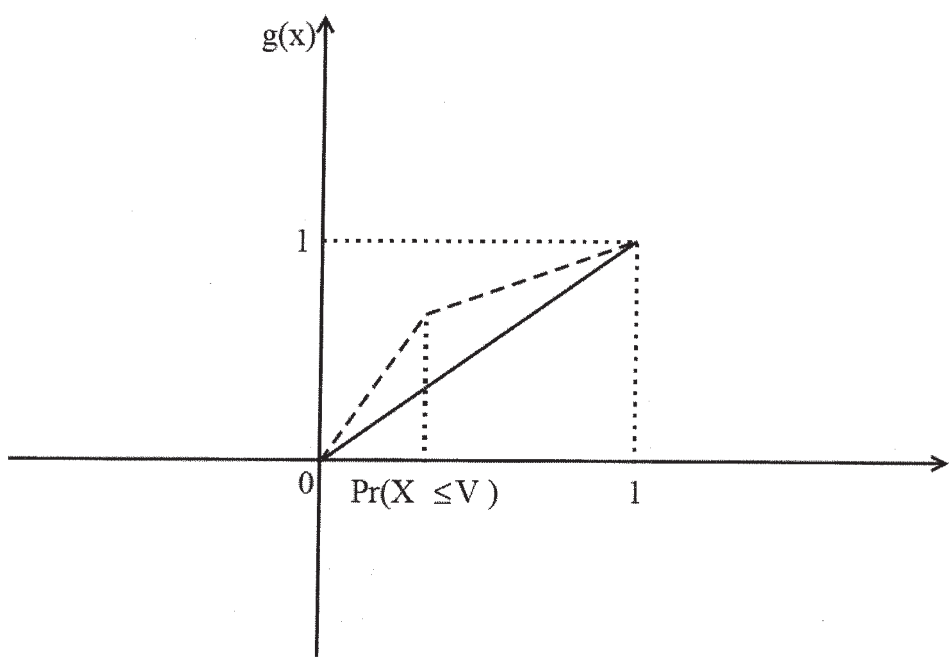

Fig.1: Function $g(x)$

Using this function $g(x)$, RDEU will be distribution dependent. The different slopes in the two classes of outcomes depend on the psychological impact of overperformance and underperformance outcomes compared to the benchmark.

Subsequently, we consider the particular case of Yaary's dual theory (DEU) for RDEU with $u(x)=x$. The risk-aversion is represented by the curvature of *g(p)*, rather than by the curvature of the utility function. In Yaari's theory, attitudes toward risk are characterised by a distortion applied to probability distribution function [18] . A concave *g(p)* weights low-rank outcomes more, just as a concave utility function weights low-rank outcomes more heavily.

In the portfolio theory the standard risk measure is given by

$$\varphi(X) = u[E(X)] - E[u(X)]$$

where $E(X)$ is the expectation of the random variable X and $u(x)$ is the utility function.

According to Jensen's inequality, $\varphi(X) \geq 0$, for a risk adverse DM.

Let χ be the set of portfolios. According to classical theory, we can represent each portfolio return X in the risk-return graph as a point with coordinates $(\varphi(X), E(X))$. First, the DM determines an opportunity frontier β, given by the solution to

(P1)
$$\begin{aligned} &\min \quad \varphi(X) \\ &s.t. \\ &E(X) = m \\ &X \in \chi \end{aligned}$$

Then, the DM determines the efficient frontier ε as a solution of

(P2)
$$\begin{aligned} &max \quad E(X) \\ &s.t. \\ &\varphi(X)=\overline{\varphi} \\ &X \in \beta \end{aligned}$$

The optimal portfolio is given by the point of tangency between the efficient frontier and the expected iso-utility curves.

The DM's risk aversion enters both in the determination of ε and in the selection of optimal portfolio and is easily measurable by the Arrow-Pratt coefficient, provided that the utility function is fixed.

Even when the expected utility function is of dual type, one can define the riskness connected to each outcome X through

$$\varphi(X)=E(X)-DEU(X).$$

According to classical theory, we can determine β and ε solving (P1) and (P2). Since risk aversion is included in the two coefficients, A and B, that modify the probability of outcomes, both $\varphi(X)$ and the optimal alternative strictly depend on the risk aversion. However, in this context finding an analytical solution for the risk aversion measure is not trivial.

For a degenerate random variable X, with dual expected utility proposed, we have

$$DEU[X]=X \text{ and } \varphi(X)=0.$$

2. THE EFFICIENT FRONTIER IN THE DUAL EXPECTED UTILITY

Consider a market with N assets. We denote by

- $\widetilde{R}_i$, $i = 1 \ldots N$, the random variable representing the return rate of the asset i in $[0, T]$;
- α_i , $i = 1 \ldots N$, the percentage of initial wealth invested in asset i;
- $\widetilde{R}_P = \sum_{i=1}^{N} \alpha_i \widetilde{R}_i$, the random variable representing the return rate of the portfolio in $[0, T]$.

We assume that the expected value of a random variable can be approximated by the average of the historical data. Let R_{it} be the return of the asset i, observed during the period t, $t = 1 \ldots M$, we have $R_{Pt} = \sum_{i=1}^{N} \alpha_i R_{it}$ and

$$E(\widetilde{R}_P) = \frac{1}{M} \sum_{t=1}^{M} \alpha_i R_{it}$$

With DEU the risk measure is

$$\varphi = E(\widetilde{R}_P) - \left\{ AE\left[(\widetilde{R}_P - V)^+ \right] + AV \Pr(\widetilde{R}_P > V) + BE\left[(\widetilde{R}_P - V)^- \right] + BV \Pr(\widetilde{R}_P \leq V) \right\}$$

where

$$(\widetilde{R}_P - V)^+ = \max\{\widetilde{R}_P - V, 0\}$$

and

$$(\widetilde{R}_P - V)^- = \min\{\widetilde{R}_P - V, 0\}.$$

Since $A = \dfrac{1 - B\Pr\left(\widetilde{R}_P \le V\right)}{1 - \Pr\left(\widetilde{R}_P \le V\right)}$ and $\Pr(\widetilde{R}_P > V) = 1 - \Pr(\widetilde{R}_P \le V)$ we have

$$\varphi = E\left(\widetilde{R}_P\right) - \left\{ AE\left[\left(\widetilde{R}_P - V\right)^+\right] + BE\left[\left(\widetilde{R}_P - V\right)^-\right] + V \right\}$$

Being

$$\left(\widetilde{R}_P - V\right)^+ + \left(\widetilde{R}_P - V\right)^- = \widetilde{R}_P - V$$

and

$$-\left(\widetilde{R}_P - V\right)^+ = \left(V - \widetilde{R}_P\right)^-$$

we can write

$$\varphi = E\left(\widetilde{R}_P\right) - \left\{ AE\left[\widetilde{R}_P - V - \left(\widetilde{R}_P - V\right)^-\right] + BE\left[\left(\widetilde{R}_P - V\right)^-\right] + V \right\} =$$

$$= \left(E\left(\widetilde{R}_P\right) - V\right) + AE\left(V - \widetilde{R}_P\right) + (B - A)E\left[\left(V - \widetilde{R}_P\right)^+\right].$$

The optimisation problem (P1) can now be expressed as:

(P3) $$\min_{\alpha_1, \alpha_2, \ldots \alpha_N} \left(E\left(\widetilde{R}_P\right) - V\right) + AE\left(V - \widetilde{R}_P\right) + (B - A)E\left[\left(V - \widetilde{R}_P\right)^+\right]$$

$$\begin{aligned} & s.t. \\ & E\left(\widetilde{R}_P\right) \ge m \\ & \sum_{i=1}^{N} \alpha_i = 1 \\ & \alpha_i \ge 0 \quad \forall i = 1, \ldots, N \end{aligned}$$

where

- the first constraint is related to the minimal rate of return required by the investor;
- the second constraint states that the total sum invested in the portfolio has to be equal to the available capital ;
- the third constraints establishes that short sales are not allowed .

For each value of the vector $(\alpha_1, \alpha_2, ..., \alpha_N)$ the portfolio return is caracterized by its own distribution. Therefore, when the benchmark is exogenously fixed and there is no information with regard to the distribution of the portfolio return, the dual expected utility is distribution dependent.

Such dependency can be removed assuming the stock returns and consequently the portfolio returns have a well–known distribution and the benchmark is dependent from the returns.

In that case introducing the variables

$$z_t = \max\left\{0, V - \sum_{i=1}^{N} \alpha_i R_{it}\right\},$$

problem (P3) is equivalent to the linear problem:

(P4)

$$\min\left(\frac{1}{M}\sum_{t=1}^{M}\sum_{i=1}^{N}\alpha_i R_{it} - V\right) + \frac{A}{M}\sum_{t=1}^{M}\left(\sum_{i=1}^{N}\alpha_i R_{it} - V\right) + \frac{B-A}{M}\sum_{t=1}^{M} z_t$$

$$
\begin{aligned}
& s.t. \\
& \frac{1}{M}\sum_{t=1}^{M}\sum_{i=1}^{N}\alpha_i R_{it} \geq m \\
& z_t \geq V - \sum_{i=1}^{N}\alpha_i R_{it} \\
& z_t \geq 0 \\
& \sum_{i=1}^{N}\alpha_i = 1 \\
& \alpha_i \geq 0 \quad \forall i = 1,\ldots,N.
\end{aligned}
$$

where $\frac{1}{M}$ is the probability, common to all the realisation of the random variable R_{it}, associated to the rate of return of the portfolio at time t.

An optimal solution $\left(\alpha_1^*,\ldots\alpha_N^*, z_1^*,\ldots,z_M^*\right)$ for (P4) satisfies the conditions

$$
z_t^* = \max\left\{0, V - \sum_{i=1}^{N}\alpha_i R_{it}\right\}
$$

so it is also an optimal solution for (P3).

The main advantage of our model is that it allows to find the optimal portfolio by solving the following linear problem

(P5)
$$
\max \frac{A}{M}\sum_{t=1}^{M}\left(V - \sum_{i=1}^{N}\alpha_i R_{it}\right) - \frac{B-A}{M}\sum_{t=1}^{M} z_t + V
$$

$$
\begin{aligned}
& s.t. \\
& z_t \geq V - \sum_{i=1}^{N}\alpha_i R_{it} \\
& z_t \geq 0 \\
& \sum_{i=1}^{N}\alpha_i = 1 \\
& \alpha_i \geq 0 \quad \forall i = 1,\ldots,N.
\end{aligned}
$$

With this DEU, the DM optimum portfolio is chosen by minimising the negative distance from the portfolio return and the benchmark and maximising the positive distance from the portfolio return and the benchmark.

If $V = E(\widetilde{R}_P)$, and $\widetilde{R}_P$ is normally distributed: $\Pr(\widetilde{R}_P \leq V) = \frac{1}{2}$, $A = 2 - B$ so that in (P4) the objective function is

$$(2B-2)E\left[\left(V-\widetilde{R}_p\right)^+\right]$$

which is the product of the usual measure of downside risk and (2B-2).

In this case, DEU is a linear combination of portfolio expected return m and standard deviation σ (For the proof see Appendix A).

$$DEU = m - \frac{\sigma}{\sqrt{2\pi}}(2B-2).$$

3. SOME REMARKS ABOUT THE PARAMETERS

The DEU considered here allows the probabilities of the random variable to be rescaled on the basis of the DM's risk aversion that determines the value B.

We can assume that B-A is a measure of the decision maker's risk aversion: in the case of a risk-adverse DM, this quantity is always ≥ 0 and its value increases as the weight assigned to the average of outcomes greater than the benchmark decreases.

Note for A = B=1, the individual has no perception of the risk connected with the portfolio randomness, since the risk measure associated to portfolio return vanishes.

When we choose the benchmark so that we know $\Pr(\tilde{R}_P \le V)$, the risk aversion is included in the only parameter B being

$$A = \frac{1 - B\Pr(\tilde{R}_P \le V))}{1 - \Pr(\tilde{R}_P \le V)}. \tag{1}$$

Furthermore, (1) implies that A = 0 when $B = \frac{1}{\Pr(\tilde{R}_P \le V)}$, independent of $\Pr(\tilde{R}_P \le V)$. In this case, the proposed model minimizes the portfolio returns that underperform the benchmark and the risk measure is the usual downside risk measure.

4. APPLICATION

Here, we apply the model we propose in order to select the equity portfolio from the Italian stock market, included in the S&P/MIB index[1].

The monthly returns were computed on the monthly med stock prices from June 2002 to May 2005 (source: Bloomberg).

Assuming that short positions are not allowed, the monthly returns we obtain do not allow a monthly portfolio return exceeding 3.29%. The following results have been determined assuming the returns are normally distributed and the benchmark is the average value of the portfolio return so DEU is distribution-independent.

For a better understanding of the risk-aversion role in the portfolio selection process, we performed two analyses:

1) in the first step we determine the efficient frontiers ε corresponding to different levels of the B parameter;

[1] In appendix B we show the stocks considered.

2) in the second step we analyse the composition of optimal portfolios varying the DM risk aversion

Fig 2-6 show the efficient frontiers for different values of B parameter. In each plot, the values on the y axis are the expected portfolio returns while the ones on the x axis represent the associated portfolio risks.

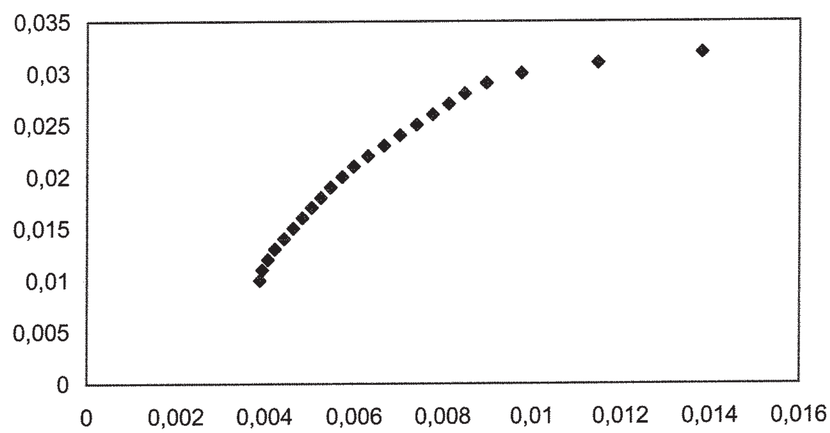

Fig. 2: The efficient frontier for A=0,8 and B=1,2.

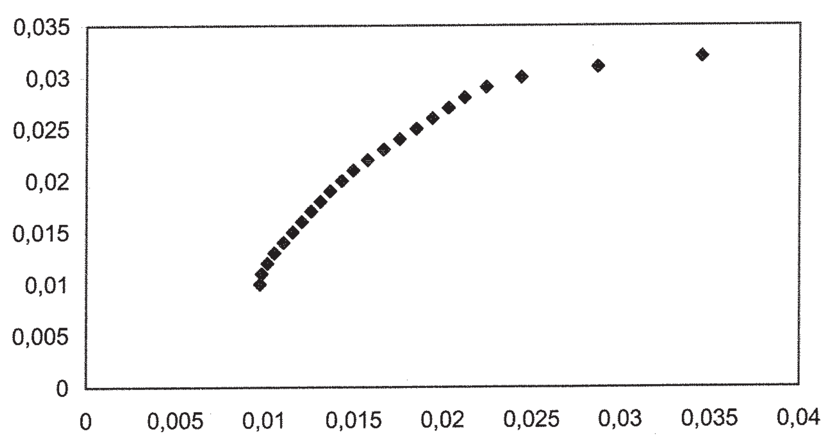

Fig.3 : The efficient frontier for A=0,5 and B=1,5.

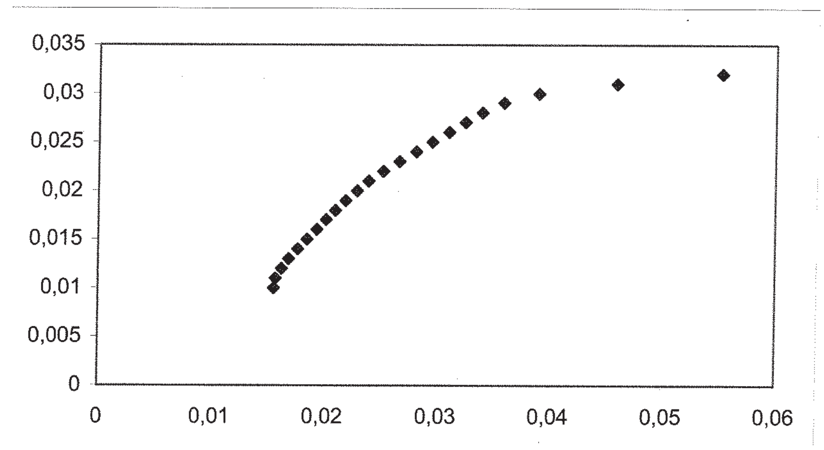

Fig.4 : The efficient frontier for A=0,2 and B=1,8.

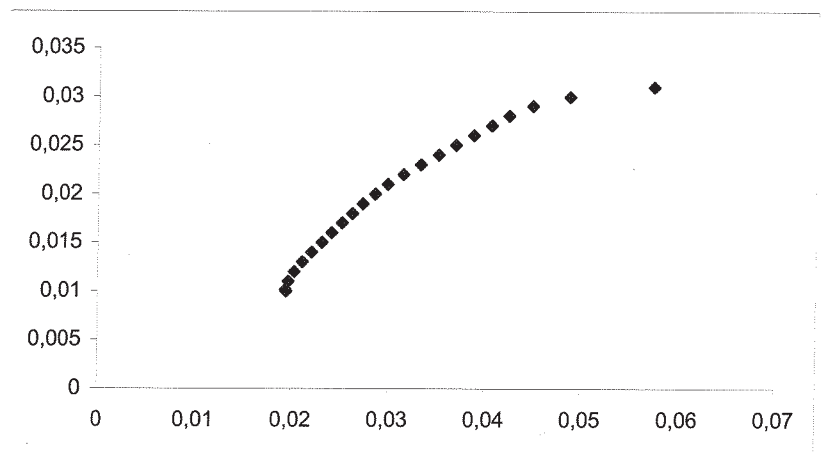

Fig.5 : The efficient frontier for A=0 and B=2.

Note that the efficient frontier curves are very similar. Obviously, for a given expected return, the risk associated with each efficient portfolio is different, since it depends on the decision maker's risk aversion.

The following table summarizes the results obtained for the optimal portfolios corresponding to different choices of B

	B=1	B=1,2	B=1,5	B=1,8	B=2
Return	0,032902	0,029714	0,028162	0,018781	0,013323
Riskiness	0	0,009457	0,021349	0,021655	0,021416
DEU	0,032902	0,020257	0,006813	-0,00287	-0,00809

Tab. 1: Optimal portfolios

Tab.1 shows that, when DM's risk-aversion grows (B grows), the individual prefers lower returns in order to reduce risk. When B=1 the DM doesn't perceive the risk and he chooses maximizing the expected return.

In Fig.6, we report the behaviour of the dual expected utility for each column of Tab.1.

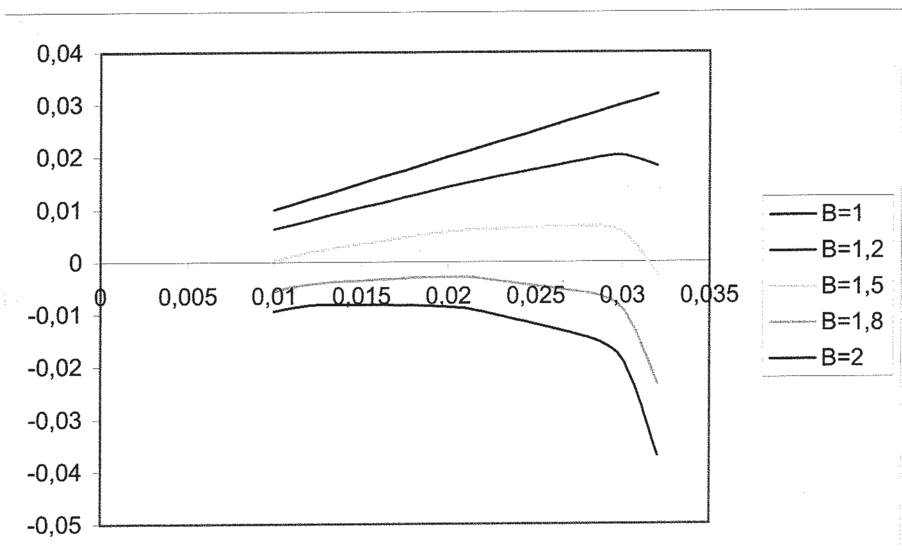

Fig.6 : Dual Expected Utility values with different risk aversion varying the expected returns

In the first three cases, the expected utility associated with the optimal portfolio exceeds the value that one would get for a risk-free investment with a monthly return of 0,00173 (corresponding to a nominal annual return of 2,1%, the current level for risk-free return area Euro). For these cases, the individual will choose to invest in the equity market.

In the other cases, if the DM compares the optimal DEU with the DEU of risk-free investment he rejects the portfolio investment.

In Tab. 2, we have the composition of optimal portfolios for B varying from 1 to 2.

We can observe that when B grows, the diversification of optimal portfolio also grows.

5. CONCLUSIONS

The proposed model for portfolio selection is subjective in nature, because in order to implement it, we not only need objective data (the time series of returns), but also subjective data such as:

- the benchmark
- the parameters that model the DM's risk-aversion.

The application of the model is interesting when the benchmark is chosen in that $\Pr(X \le V)$ is known. In this cases the model is represented by a concave increasing piece-wise linear utility function and it can be solved by linear programming.

The linearity of the model is computationally attractive, since it is easy to solve a linear program even when N is very high.

However, a direct comparison between the general DEU model and Markowitz's one is not possible. When we assume the equity return normally distributed and we choose the benchmark equal to the average portfolio's return, the efficient frontier is the same as for mean-variance models, but for more our model allows to find endogenously the optimal portfolio for each value of B:

Hadar e Kun Seo [5] demonstrate that dual expected utility in the portfolio selection allows the diversification; otherwise in our model the benchmark has a fundamental role for the diversification. In fact when the benchmark is exogenously fixed and all the data are found over the benchmark, we can obtain non-diversified portfolios. In this case, the model doesn't take risk into consideration and it becomes the maximisation of portfolio expected return.

In general, transaction costs and minimum transaction lots associated with purchasing a new portfolio cannot be neglected. In future work we wish to extend the model to allow for stock transaction costs and minimum transaction lots.

		AL	AGL	AUTO	NTV	BFI	BIN	BPM	BPVN	BNL	BUL	CAP	ENEL	ENI	FWB	F	FNC	G
B=1		0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
B=1,2		0	0	0,659457599	0	0	0	0	0	0	0	0,257218	0	0	0	0	0	0
B=1,5		0	0	0,659457599	0	0	0	0	0	0	0	0,257218	0	0	0	0	0	0
B=1,8		0	0	0,330261053	0,051343	0	0	0,071613	0	0	0	0,04072	0	0	0	0	0	0
B=2		0	0	0,040595375	0,139403	0	0	0,016585	0	0	0	0,054438	0,023982	0,200275	0	0	0	0
	ES	IT	LUX	MS	MB	MED	MN	BMPS	PC	R	RCS	SPM	SPI	SRG	STM	TIT	TIM	UC
B=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B=1,2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B=1,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
B=1,8	0	0	0	0	0	0	0	0	0	0	0	0	0	0,506063	0	0	0	0
B=2	0	0	0	0	0	0	0	0	0	0	0	0	0	0,524722	0	0	0	0

Tab. 2: Optimal portfolio composition

APPENDIX A

We assume the portfolio return is a continuous random variable $\tilde{R}_P$ normally distributed $\forall(\alpha_1, \alpha_2, ...\alpha_N)$, let m be the average and σ the standard deviation. We also assume the benchmark $V = m$

In this case

$$DEU = \frac{A}{\sqrt{2\pi}\sigma}\int_{m}^{+\infty} x \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)dx + \frac{B}{\sqrt{2\pi}\sigma}\int_{-\infty}^{m} x \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)dx$$

Assuming

$u = \frac{x-m}{\sigma}$, we have that when $x = m, u = 0$ and $dx = \sigma\, du$.

and

$$DEU = \frac{A}{\sqrt{2\pi}}\int_{0}^{+\infty}(u\sigma + m)\exp\left(-\frac{u^2}{2}\right)du + \frac{B}{\sqrt{2\pi}}\int_{-\infty}^{0}(u\sigma + m)\exp\left(-\frac{u^2}{2}\right)du =$$

$$= \frac{A\sigma}{\sqrt{2\pi}} + m\frac{A}{2} - \frac{B\sigma}{\sqrt{2\pi}} + m\frac{B}{2} = \frac{m}{2}(A+B) - \frac{B-A}{\sqrt{2\pi}}\sigma .$$

c.v.d.

APPENDIX B

Stock list

ALLEANZA ASSICURAZIONI
AUTOGRILL SPA
AUTOSTRADE SPA
BANCA ANTONVENETA SPA
BANCA FIDEURAM SPA
BANCA INTESA SPA
BANCA POPOLARE DI MILANO
BANCO POPOLARE DI VERONA E N
BANCA NAZIONALE LAVORO-ORD
BULGARI SPA
CAPITALIA
ENEL SPA
FASTWEB
FIAT SPA
FINMECCANICA SPA
ASSICURAZIONI GENERALI
GRUPPO EDITORIALE L'ESPRESSO
ITALCEMENTI SPA
LUXOTTICA GROUP SPA
MEDIASET SPA
MEDIOBANCA SPA
MEDIOLANUM SPA
ARNOLDO MONDADORI EDITORE
BANCA MONTE DEI PASCHI SIENA
PIRELLI & C.
RAS SPA
RCS MEDIAGROUP SPA
SAIPEM
SANPAOLO IMI SPA
SNAM RETE GAS
STMICROELECTRONICS
TELECOM ITALIA SPA
TIM SPA
UNICREDITO ITALIANO SPA

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