Mathematical Methods for Economics (60h – 9CFU)

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Course learning objectives and skill acquisition

The aim of the course is to consolidate and deepen the mathematical method as a fundamental investigation tool for economic, financial and business disciplines. In particular, problems of function optimization will be studied. To this end the course is divided into two parts. The first part will introduce the fundamental concepts of the analysis of functions in several real variables and developed the tools to recognize and study free and constrained static optimization problems, both the local and global problem. The second part deals with dynamical models, studying in particular problems of dynamic optimization in discrete time.

Assessment

The course assessment is based on a final exam with a written and an oral part, closed books.

Course general schedule

1. Tuesday 15:00-17:00
2. Wednesday 12:30-14:30
3. Friday 8:30-10:30

Teaching material

Teaching material will be available to students through the Moodle platform, in the form of problem set, sample exams and recording of lectures.

Textbooks

- **Simon C.P. & Blume L.** "Mathematics for Economists" (1994), ed. Norton & Co: from ch. 12 to ch. 22.
- **Salsa S. & Squellati A.** "Dynamical systems and optimal control" (2018), ed. Egea-Bocconi University Press: ch. 1 (pp.1-5, 17-24), ch 3 (pp.65-69, 74-75), ch. 9, ch 10 (pp. 239-243, 270-285); ch. 11.

Detailed teaching agenda

Basics on one-variable functions. Main properties and definitions. First order derivative and rules. Tangent line. Applications of one-variable differential calculus: first order derivative and monotonicity, second order derivative and convexity. Local maxima and minima: first order necessary condition (FONC) and second order necessary and sufficient conditions (SONSC). Global maximum and minimum: Weierstrass's Theorem.

Topology on R^n. One dimensional real sequences. The Euclidean and metric space R^n. Sequences in R^n. Open, closed and compact sets of R^n.

Functions of several variables. Definition and examples. Level curves. Linear functions and quadratic forms. Continuous functions and Weierstrass theorem (w.p.).

Differential calculus for functions of several variables. Partial derivatives. The differential and the tangent plane. Gradient and Jacobian matrix. Approximations using differentials. The chain rule: differentiating along a curve, directional derivatives, the general case. Higher order derivatives and the Hessian matrix.

Implicit Function Theorem: Implicit function theorem for functions of two variables. Geometric interpretation. Implicit function theorem for functions of several variables. Systems of implicit functions. The linear case: m equations, n + m variables. General systems of m equations and n + m variables. Inverse function theorem.

Static optimization. *Free optimization*. NC for the existence of local maxima and minima (w.p.). Stationary points. Quadratic forms and sign. Criterion for 2 or 3 variables. Second order SC for the existence of local maxima and minima. *Constrained optimization*. Equality constraints. NC for the existence of local maxima and minima. Inequality constraints: NC for the existence of local maxima and minima. Kuhn-Tucker conditions for non-negative variables. The meaning of the Lagrange multiplier (w.p.).

Homogeneous functions. Geometric definition and properties. Euler's theorem. Homothetic functions.

Concave and convex functions. Geometric definition. Properties and characterization. Concavity and sign of the Hessian matrix. Quasiconcave and quasiconvex functions. Cobb-Douglas functions. Pseudoconcave functions. Free and constrained optimization in the hypothesis of quasiconvexity and quasiconcavity.

Dynamic models: Discrete and continuous time Malthus model. Dynamic models in continuous and discrete time. Introduction to differential and difference equation. First order difference equations: the linear case. Existence and uniqueness of solutions.

The Calculus of Variations. An introduction. Necessary conditions in integral form (w.p.). Euler-Lagrange equation (NC). Transversality conditions. Sufficient conditions in the hypothesis of concavity or convexity.

Introduction to optimal control theory. The optimal control problem (in discrete and continuous time). Pontryagin maximum principle. Sufficient conditions in the hypothesis of concavity.

Dynamic programming. Finite horizon and discrete time case: Bellman's principle of optimality. The value function. Dynamic programming Principle. Bellman equation and optimality conditions. Infinite horizon and discounting.